Risk Aversion

A risk-indifferent individual chooses the gamble with highest expected value.
A risk-averse individual will surrender expected value for reduced risk.

Risk-Averse Behavior

- Insurance;
- Portfolio diversification;
- Bernoulli game.

Insurance

Buying insurance
- Reduces risk;
- Reduces expected value (the premium exceeds the expected value of the payout).

Portfolio Diversification

A risk-indifferent investor simply invests everything in the asset with the highest expected return.
A risk-averse investor diversifies. The diversification reduces the expected return but also reduces the risk.

Proof that Diversification Reduces Risk

Consider a two risky asset example. An investor invests the fraction $1 - f$ of his wealth in a low-risk asset, for which the standard deviation of the rate of return is one. He invests the fraction $f$ in a higher risk asset, for which the standard deviation of the rate of return is $s > 1$. The correlation between the two rates of return is $r$.

The variance of the rate of return on the portfolio is

$$\text{Var}(R_f) = (1 - f)^2 1 + f^2 s^2 + 2 (1 - f) frs.$$  

Differentiating and setting $f = 0$ gives

$$\frac{d [\text{Var}(R_f)]}{df} = (-2 + 2f) + 2fs^2 + 2 (1 - 2f) rs$$

$$= 2 (rs - 1) \text{ at } f = 0.$$  

Diversification pays if the derivative at $f = 0$ is negative. Diversification necessarily pays if $r \leq 0$. If $r > 0$, then diversification pays if and only if $r < 1/s$. 
Bernoulli Game

Consider a gamble: flip a coin until it comes up tails; then the game ends.

If the first tail is on the $n$th flip, you win $2^n$ dollars.

How much would you pay to play the game one time?

A risk indifferent individual would pay any amount!

The expected value of the game is infinite. As the probability of the first tail occurring on the $n$th flip is $1/2^n$, the expected value is

$$
\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots
$$

$$
= 1 + 1 + 1 + \cdots
$$

$$
= \infty.
$$

If an individual refuses to risk everything he owns to play the game, then he must be risk averse.