Risk Aversion

A risk-indifferent individual chooses the gamble with highest expected value.

A risk-averse individual will surrender expected value for reduced risk.

Risk-Averse Behavior

- Insurance;
- Portfolio diversification;
- Bernoulli game.

Insurance

Buying insurance

- Reduces risk;
- Reduces expected value (the premium exceeds the expected value of the payout).

Portfolio Diversification

A risk-indifferent investor simply invests everything in the asset with the highest expected return.

A risk-averse investor diversifies. The diversification reduces the expected return but also reduces the risk.

Proof that Diversification Reduces Risk

Consider a two risky asset example. An investor invests the fraction $1 - f$ of his wealth in a low-risk asset, for which the standard deviation of the rate of return is one. He invests the fraction f in a higher risk asset, for which the standard deviation of the rate of return is $s > 1$. The correlation between the two rates of return is r.

The variance of the rate of return on the portfolio is

$$\operatorname{Var}\left(R_f\right) = (1-f)^2 1 + f^2 s^2 + 2(1-f) f r s.$$

Differentiating and setting $f = 0$ gives

$$\begin{aligned}\frac{\mathrm{d}\left[\operatorname{Var}\left(R_f\right)\right]}{\mathrm{d}f} &= (-2+2f) + 2fs^2 + 2(1-2f)rs \\ &= 2(rs-1) \text{ at } f = 0.\end{aligned}$$

Diversification pays if the derivative at $f = 0$ is negative.

Diversification necessarily pays if $r \leq 0$.

If $r > 0$, then diversification pays if and only if $r < 1/s$.

Bernoulli Game

Consider a gamble: flip a coin until it comes up tails; then the game ends.

If the first tail is on the nth flip, you win 2^n dollars.

How much would you pay to play the game one time?

A risk indifferent individual would pay any amount!

The expected value of the game is infinite. As the probability of the first tail occurring on the nth flip is $1/2^n$, the expected value is

$$\begin{aligned}\left(\frac{1}{2}\right)2^1 + \left(\frac{1}{2}\right)^2 2^2 + \left(\frac{1}{2}\right)^3 2^3 + \cdots \\ = 1 + 1 + 1 + \cdots \\ = \infty.\end{aligned}$$

If an individual refuses to risk everything he owns to play the game, then he must be risk averse.