Risk Aversion

A risk-indifferent individual chooses the gamble with highest expected value.

A risk-averse individual will surrender expected value for reduced risk.
Risk-Averse Behavior

- Insurance;
- Portfolio diversification;
- Bernoulli game.
Insurance

Buying insurance

- Reduces risk;
- Reduces expected value (the premium exceeds the expected value of the payout).
Portfolio Diversification

A risk-indifferent investor simply invests everything in the asset with the highest expected return.

A risk-averse investor diversifies. The diversification reduces the expected return but also reduces the risk.
Proof that Diversification Reduces Risk

Consider a two risky asset example. An investor invests the fraction $1 - f$ of his wealth in a low-risk asset, for which the standard deviation of the rate of return is one. He invests the fraction $f$ in a higher risk asset, for which the standard deviation of the rate of return is $s > 1$. The correlation between the two rates of return is $r$. 
The variance of the rate of return on the portfolio is

$$\text{Var} \left( R_f \right) = \left(1 - f\right)^2 1 + f^2 s^2 + 2 (1 - f) f r s.$$ 

Differentiating and setting $f = 0$ gives

$$\frac{d}{df} \left[ \text{Var} \left( R_f \right) \right] = (-2 + 2f) + 2fs^2 + 2 (1 - 2f) rs$$

$$= 2 (rs - 1) \text{ at } f = 0.$$ 

Diversification pays if the derivative at $f = 0$ is negative.

Diversification necessarily pays if $r \leq 0$.

If $r > 0$, then diversification pays if and only if $r < 1/s$. 

6
Bernoulli Game

Consider a gamble: flip a coin until it comes up tails; then the game ends.

If the first tail is on the $n$th flip, you win $2^n$ dollars.

How much would you pay to play the game one time?
A risk indifferent individual would pay any amount!

The expected value of the game is infinite. As the probability of the first tail occurring on the $n$th flip is $1/2^n$, the expected value is

$$\left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \cdots$$

$$= 1 + 1 + 1 + \cdots$$

$$= \infty.$$ 

If an individual refuses to risk everything he owns to play the game, then he must be risk averse.