Erratic Price, Smooth Dividend

Shiller [1] argues that the stock market is inefficient: stock prices fluctuate too much.

According to economic theory, the stock price should equal the present value of expected dividends. However dividends are very stable; they fluctuate very little about an upward trend. Expected dividends should therefore also fluctuate little, and consequently stock prices should be stable.

In fact, stock prices fluctuate wildly.

Variance Bound

Shiller shows how the variability of the dividend sets an upper bound to the variability of the stock price.

Present Value

Let p_t denote the stock price at time t, and let d_t denote the dividend during period t (the dividend is actually paid at time $t+1$). According to economic theory, the price is the present value of expected dividends,

$$p_t = \frac{\mathrm{E}(d_t)}{1+r} + \frac{\mathrm{E}(d_{t+1})}{(1+r)^2} + \cdots . \tag{1}$$

Ex Post Rational Price

Define the "*ex post* rational price" p_t^* as the present value of *actual* dividends,

$$p_t^* = \frac{d_t}{1+r} + \frac{d_{t+1}}{(1+r)^2} + \cdots . \tag{2}$$

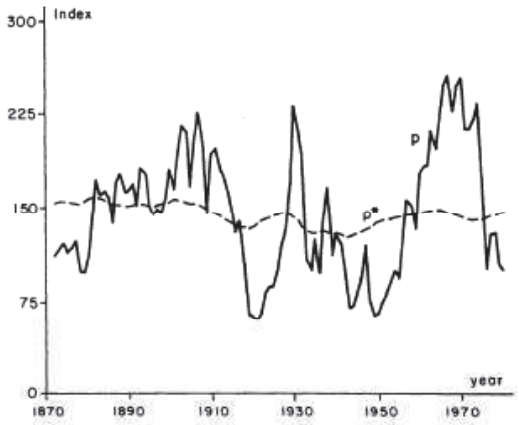
Economic theory distinguishes between *ex post*—meaning "after"—and *ex ante*—meaning "before." The *ex ante* (rational) price is just the actual price, based on the present value of expected dividends.

Standard and Poor's Composite Stock-Price Index

In figure 1[1, p. 422], the price is the Standard and Poor's Composite Stock-Price Index for 1871-1979, detrended by an exponential growth factor. The *ex post* rational price is calculated from the dividend.

Whereas the *ex post* rational price is stable, the actual price is erratic.

Figure 1: Standard and Poor's Composite



Forecast

By (1) and (2), the price is the forecast of the *ex post* rational price,

$$p_t = \mathrm{E}(p_t^*).$$

We write

$$p_t^* = p_t + e_t, \tag{3}$$

in which e_t denotes the forecast error.

Rational Forecasting

Consider a scatter diagram of the *ex post* rational price plotted against the price. If forecasting is rational, the data are scattered randomly about a 45° line from the origin.

A rational forecast (the optimum forecast based on all available information) has the following two properties:

- The error should have mean zero.
- The error should be uncorrelated with the forecast.

If the mean were positive, then one could improve the forecast just by increasing all forecast values. If the correlation were positive, then one could improve the forecast by increasing high forecast values and decreasing low forecast values.

Variance Decomposition

In (3), p_t and e_t are uncorrelated. Consequently

$$\mathrm{Var}(p_t^*) = \mathrm{Var}(p_t) + \mathrm{Var}(e_t), \tag{4}$$

since the variance of the sum of two uncorrelated random variables is the sum of the variances. Expression (4) decomposes the variance of the *ex post* rational price into the variance of the forecast and the variance of the error.

Variance Bound

The inequality

$$\mathrm{Var}(p_t^*) \geq \mathrm{Var}(p_t)$$

is a *variance bound*. The variance of the *ex post* rational price is an upper limit (upper bound) on the variance of the price.

Variance-Bound Test

Unfortunately the variance bound is violated. Figure 1shows that the variance of the price is *much* larger than the variance of the *ex post* rational price.

Hence the stock market is inefficient.

Dividend Yield

This inefficiency means that one can forecast the rate-of-return on stocks from the dividend yield (the dividend/price ratio).

A profitable trading rule is to buy when the dividend yield is high (because the price is then too low) and to sell when the dividend yield is low (because the price is then too high).

Long-Run Autocorrelation

For some investment, let x_t denote the total excess holding-period return from time t to time $t+T$. If the market is efficient, the correlation of x_t and x_{t-T} must be zero.

Many successful tests of the random-walk theory test for a zero correlation when T is short, such as a day or a week.

Long Period

When T is long (such as five years), however, one finds a negative correlation. Thus a low return tends to be followed by a high return, and *vice versa*.

Statistical Problem

This negative correlation is not, however, statistically significantly different from zero, because the number of independent observations is small.

For Shiller's data period 1871-1979, the 5-year time period means that the 109 years of data amount to only 22 independent observations. With so few observations, one cannot reject a zero correlation with any confidence.

It is possible to show that this same problem also afflicts the forecasting of the rate of return from the dividend yield.

Influence of the Great Depression

In the Great Depression, stock prices fell greatly but ultimately bounced back. This price movement was so great that this single episode dominates statistical testing. Omitting the period of the Great Depression eliminates the negative correlation of long-run returns.

References

[1] Robert J. Shiller. Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review*, 71(3):421–436, June 1981. HB1E26. 1, 5