Statistical Testing of the Random-Walk Theory

The random-walk theory of stock prices is the best-tested and best-verified theory in economics!

Many statistical tests support the random-walk theory.

Difficulty of Statistical Testing

Most theories in economics are difficult to test with data.

The key problem is that typically a controlled experiment is not possible. The economist makes inferences from observed data.

(Astronomy shares this trait with economics.)

Ceteris Paribus

Economic theory postulates a relationship between two economic variables, with other things held constant. Yet in practice many things change simultaneously, and the alleged relationship cannot be isolated.

Downward-Sloping Demand?

A simple example is the theory of demand. Although all economists believe that the demand curve for a good is downward-sloping, to establish this relationship by data is extremely difficult.

Success

Perhaps the success of the statistical testing of the random-walk theory is that the theory applies regardless of whether other things change.

Surprising Result

This success has surprised many people, as many have believed that skillful use of technical analysis (charting) allows one to make economic profits.

Statistical Testing

The random-walk theory asserts that there is no pattern to stock-price changes. In particular, past stock-price changes do not enable one to predict future price changes.

One tests the theory by investigating whether any forecasting is possible. Do past stock-price changes enable one to forecast future price changes? Even a small ability to forecast would contradict the random-walk theory.

Graph of Stock Prices

A simple non-statistical test is just to graph a stock price as a function of time. The jagged appearance of the graph conforms with the random-walk theory. As the price change at one moment is uncorrelated with past price changes, the incessant up-and-down movement makes the graph jagged.

If the graph were smooth, this finding would contradict the random-walk theory. A movement up or down would continue, perhaps only for a brief time, but this continuation would create an opportunity for economic profit.

Correlation

A simple statistical test of the random walk theory is to calculate the correlation of the stock-price change during a period with the stock-price change during a previous period. For example, one can calculate the correlation of the daily stock-price change with the change on the previous day, or with the change two days ago.

The random-walk theory says that this correlation must be zero. Any non-zero correlation would allow one to forecast the future stock-price change somewhat, and one could make an economic profit.

Sample Correlation

One tests the theory by calculating the sample correlation for stock-price changes.

A statistical test allows for possible random variation in the data. If the sample correlation is far from zero, one infers that the random-walk theory is probably wrong, as this value is unlikely to occur by chance. If the sample correlation is near zero, then the data are consistent with the theory.

Probability Distribution of the Sample Correlation

If the true correlation is zero, the probability distribution of the sample correlation is approximately normal, with mean zero. The standard deviation of the sample correlation is

$$\frac{1}{\sqrt{n}},$$

in which n is the number of observations.

Critical Value

The *critical value* refers to the borderline value for accepting or rejecting the null hypothesis that the random-walk theory is true.

If the random-walk theory is valid, then 95% of the time the sample correlation will lie within 1.96 standard deviations of zero.

The critical value is therefore

$$\pm\frac{1.96}{\sqrt{n}}.$$

One rejects the random walk if the sample correlation lies away from zero by more than this critical value. An increase in the number of observations reduces the critical value.

Example

For $n = 400$, then

$$\frac{1.96}{\sqrt{n}} = \frac{1.96}{20} = .098.$$

Accept

If the sample correlation is no further than .098 from zero, then one accepts the null hypothesis that the random-walk theory is valid, as any value this near zero is compatible with the theory.

Reject

If the sample correlation is further than .098 from zero, then one rejects the null hypothesis that the random-walk theory is valid. If the random-walk theory were valid, then a value this far from zero could happen only with probability 5%, so the data suggests that the theory is wrong.

Runs Test

Another simple statistical test is a *runs test*. For daily data, a run is defined as a sequence of days in which the stock price changes in the same direction.

For example, consider the following combination of upward and downward price changes:

$$++--+-+----++.$$

A $+$ sign means that the stock price increased, and a $-$ sign means that the stock price decreased. Thus the example has 7 runs, in 12 observations.

Momentum

Momentum investing rejects the random-walk theory. The assumption is that trends continue: a price increase implies further price increases; a price decrease implies further price decreases. One buys when the stock price is rising and sells when it is falling.

Expected Number of Runs

For n observations, what is the expected number of runs?

According to the random-walk theory, the expected number of runs is

$$\frac{n}{2}.$$

Each day the probability that a new run starts is one half, and the probability that the current run continues is one half.

According to the momentum theory, runs tend to continue. Hence the expected number of runs is less.

One-Tailed Test

One tests the theory by calculating the number of runs in a data set. A one-tailed test is natural, as the momentum theory predicts fewer runs than the random-walk theory.

If the random-walk theory is true, the expected number of runs is $n/2$, and the standard deviation of the number of runs is

$$\frac{\sqrt{n}}{2}.$$

With probability 5%, the number of runs will lie more than 1.64 standard deviations below the expected value, and this number is the critical value.

Example

For $n = 400$, then

$$1.64\frac{\sqrt{n}}{2} = 16.4.$$

The expected number of runs is 200.

Hence one rejects the null hypothesis that the random-walk theory is true if the number of runs is 183 or less; this low number could occur by chance only 5% of the time.

If the number of runs is 184 or more, than one accepts the null hypothesis. This number is close enough to 200 to be compatible with the random-walk theory.

Technical Note

A possibility is that on certain days the stock price does not change. One can deal with this possibility just by ignoring the observations on these days.

Trading Rule

An implication of the efficient market/random-walk theory is that no trading rule will yield an economic profit.

Yet some authorities have proposed a mechanical trading rule.

One can test the random-walk theory by trying out the trading rule on historical data.

Statistical Testing of a Trading Rule

If the random-walk theory holds, the probability distribution of the profit from a trading rule will be random.

One can carry out a statistical test by a computer simulation.

Using a random-number generator, generate n random numbers. Using these numbers, create a random series of stock prices. Apply the trading rule to this artificial data, and calculate the profit.

Repeat this generation many times, to obtain a probability distribution for the profit, valid if the random-walk theory holds.

One then compares the profit from the trading rule to the upper 5% tail of this probability distribution. If the profit lies in this tail, then one rejects the null hypothesis that the random-walk theory is valid, as there is only a 5% chance that the profit would be so high. If the profit does not lie in the upper tail, then one accepts the null hypothesis that the random-walk theory is valid.