Term Structure

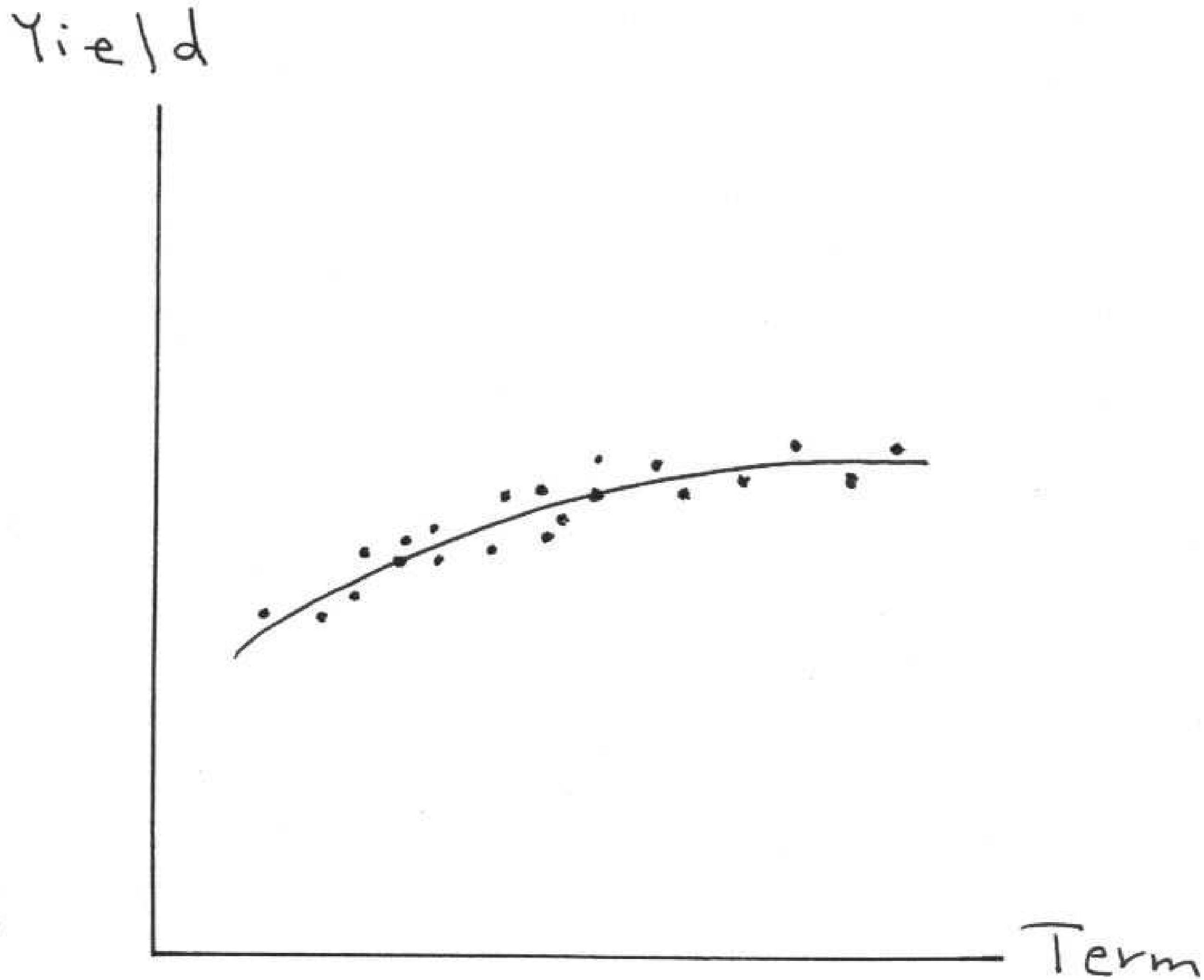
The *term structure* refers to the relationship between short-term and long-term interest rates.

Yield Curve

The *yield curve* plots the yield to maturity against the term to maturity (figure 1).

One plots a yield curve only for bonds of the “same” type. For example, one might plot the yield curve for U. S. Treasury securities.

Figure 1: Yield Curve



Observed Yield Curves

(Figures 2 and 3.)

Upward-Sloping When interest rates are low;

Flat When interest rates are medium;

Downward-Sloping When interest rates are high.

The yield curve flattens out at long maturities.

Low/medium/high are judged relative to "normal." What is seen as normal may change as time passes.

Figure 2: Yield Curve Shapes

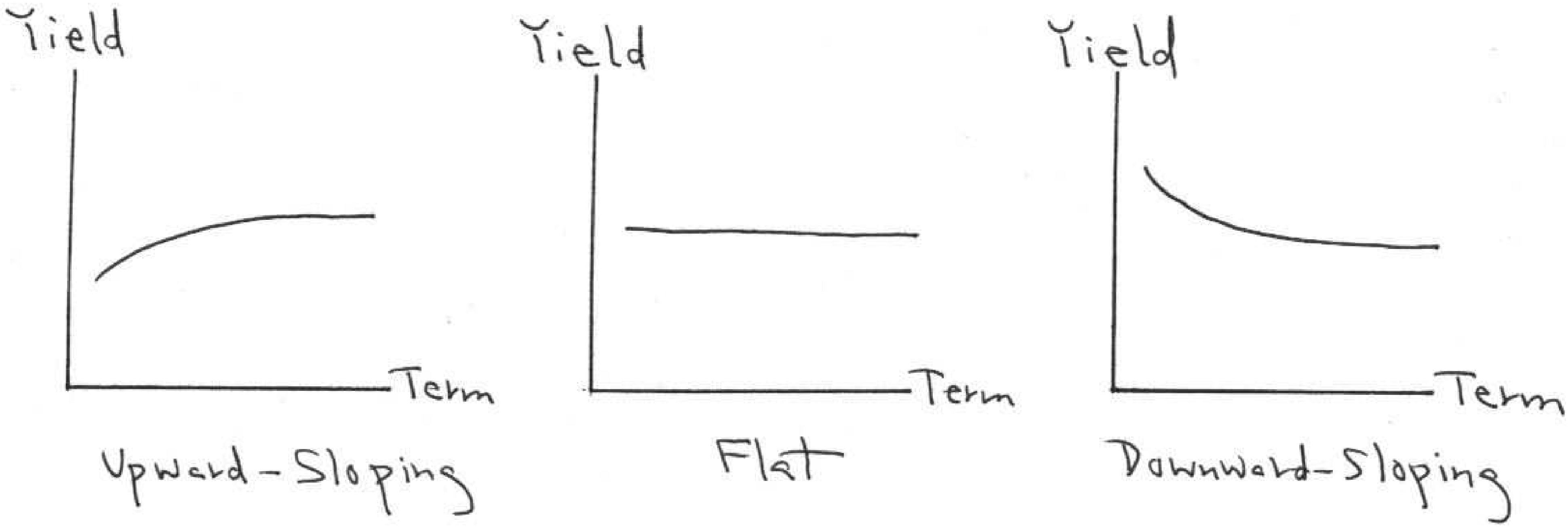
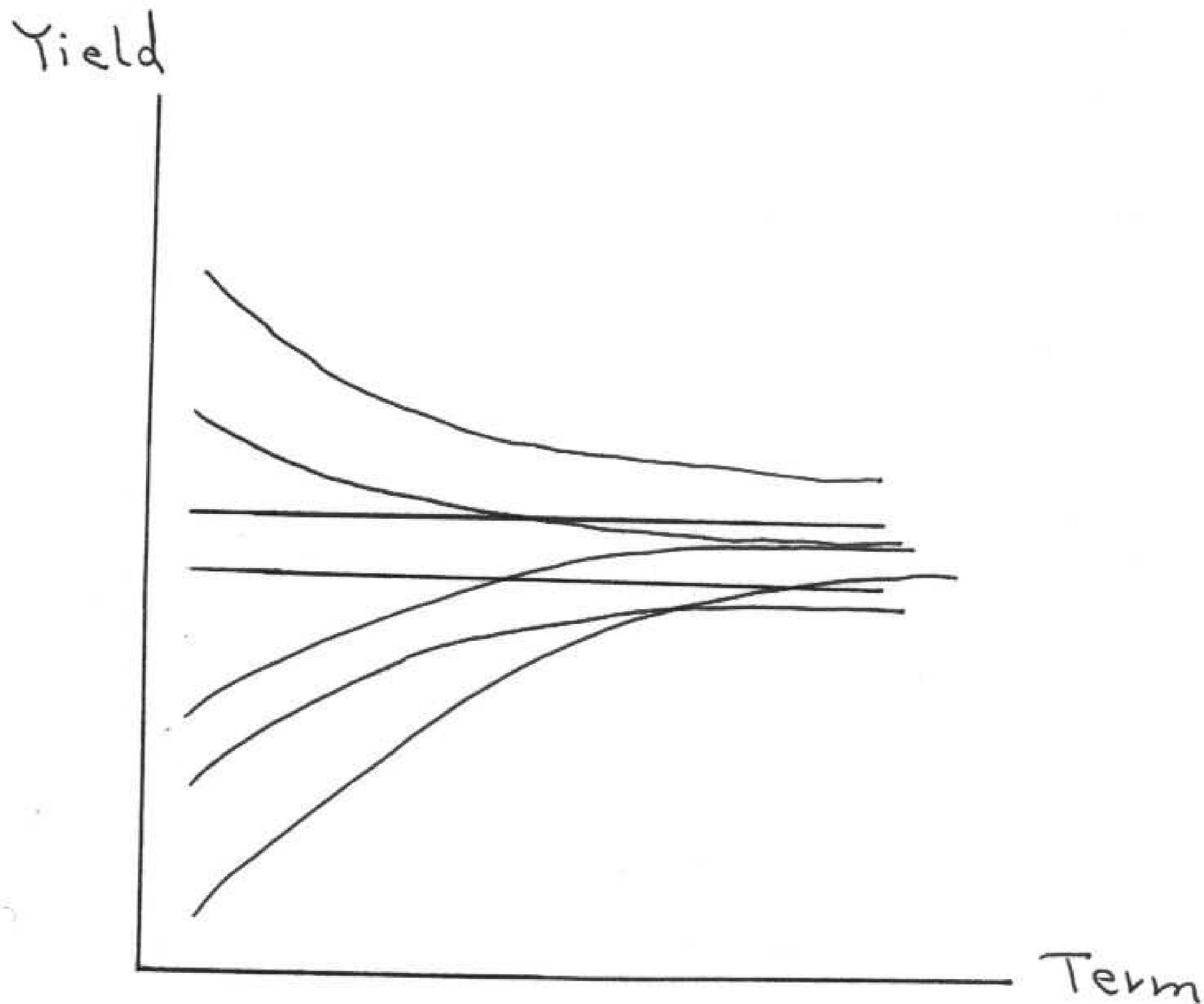


Figure 3: Yield Curves Superimposed



Implications of the Observed Yield Curves

Short-term and long-term interest rates are positively correlated.

Short-term interest rates fluctuate more than long-term interest rates.

Bond Price Fluctuation

Long-term bonds fluctuate in price by a greater percentage than short-term bonds.

The fluctuation in price is the duration times the fluctuation in the yield to maturity. That the duration is longer for long-term bonds tends to make these bonds fluctuate more in price. However, that the fluctuation in the long-term interest rate is less makes these bonds fluctuate less in price. So why does the duration effect dominate?

Expectations Theory

The expectations theory says that the long-term interest rate is the average of the current and expected future short-term rates.

For example, the yield to maturity on a five-year bond is the average of the current and expected future short-term rate for the next five years.

The expectations theory follows from the rate-of-return condition for asset-market equilibrium.

The *holding-period return* on an asset refers to the rate of return obtained by buying the asset, holding it for a period of time, and then selling it.

By the rate-of-return condition for asset-market equilibrium, short-term and long-term bonds must have the same expected holding-period return, regardless of the holding period.

Close Substitutes

The rate-of-return condition rests on the concept that different assets are *close substitutes*: investors will invest wherever the expected rate-of-return is highest.

That short-term and long-term bonds are close substitutes is the framework underlying the expectations theory. What this assumption means is that many participants in the bond market are indifferent between short-term and long-term bonds. Investors will invest wherever the expected holding-period return is higher, and bond issuers will issue bonds for which the expected holding-period return is lower.

Example

Let R_2 denote the interest rate (yield to maturity) on a two-year bond.

Let R_1 denote the interest rate on a one-year bond.

Let $\mathrm{E}(R_1)$ denote the expected interest rate next year on a one-year bond (the interest rate one year from now on a bond maturing two years from now).

The expectations theory says

$$R_2 = \frac{1}{2}\left[R_1 + \mathrm{E}\left(R_1\right)\right]. \tag{1}$$

We derive the expectations theory from the theory of asset-market equilibrium.

Two-Year Holding Period

Consider an investor with a two-year time horizon. His interest is the value of his investment in two years, and he cares about the two-year holding-period return.

An investor could simply buy a two-year bond and hold it to maturity. His total return would be

$$2R_2.$$

(For simplicity, we ignore compounding.)

Alternatively, he could buy a succession of two one-year bonds. He initially buys one-year bonds. When these bonds mature, he buys one-year bonds, maturing two years from the initial investment.

His expected total return for two years is

$$R_1 + \mathrm{E}\left(R_1\right).$$

Market Equilibrium

When one- and two-year bonds are close substitutes, then market equilibrium requires that either investment must have the same expected holding-period return:

$$2R_2 = R_1 + \mathrm{E}\left(R_1\right).$$

But this condition is just a rearrangement of the expectations theory (1).

One-Year Holding Period

Alternatively, consider an investor with a one-year time horizon. His interest is the value of his investment in one year, and he cares about the one-year holding-period return.

An investor could simply buy a one-year bond and hold it to maturity. His total return would be

$$R_1 .$$

Alternatively, he could buy a two-year bond and sell it after one year.

The total return over two years on the bond is $2R_2$. After one year, it is a one-year bond and must be priced to yield the one-year interest rate. On average, it will be priced to yield $\mathrm{E}\left(R_1\right)$ during the second year.

Hence its average holding-period return during the first year must be

$$2R_2 - \mathrm{E}\left(R_1\right).$$

Market Equilibrium

When one- and two-year bonds are close substitutes, then market equilibrium requires that either investment must have the same expected holding-period return:

$$R_1 = 2R_2 - \mathrm{E}\left(R_1\right).$$

But this condition is just a rearrangement of the expectations theory (1).

Return to Normal

We assume that people expect interest rates to return to “normal.” When interest rates are low, people expect them to be higher in the future. When interest rates are high, people expect them to be lower in the future.

Explanation of Observed Yield Curves

This assumption explains the observed yield curves.

When the short-term interest rate is low, the expected future short-term rate is higher:

$$R_1 < \mathrm{E}(R_1).$$

According to the expectations theory,

$$R_2 = \frac{1}{2}\left[R_1 + \mathrm{E}(R_1)\right] > R_1.$$

The yield curve is upward sloping.

Conversely, when the short-term interest rate is high, the expected future short-term rate is lower. The expectations theory then implies that the yield curve is downward sloping.

It follows that the short-term interest rate fluctuates more than the long-term rate.

Price Fluctuation

The expectations theory also explains why long-term bonds fluctuate more in price than short-term bonds.

Suppose that suddenly the short-term interest rate R_1 rises 1%. Since the interest rate is expected to return to normal, the expected future short-term rate $\mathrm{E}\left(R_1\right)$ also rises, but less than 1%. By the expectations theory, the two-year interest rate R_2 rises, by less than 1% but more than $\frac{1}{2}\%$.

Duration

Use duration to analyze the price change.

If the duration of the one-year bond is approximately one year, then its price falls 1%.

If the duration of the two-year bond is approximately two years, then its price falls by two times the increase in its interest rate. Since its interest rate rises by more than $\frac{1}{2}\%$ but less than 1%, its price falls *more* than 1%.

Interest-Rate Risk

Interest-rate risk has an uncertain effect on the term structure.

If an investor has a two-year time horizon, then for him the two-year bond is safe and one-year bonds are risky (the return the second year is unknown). If he is risk averse, he will buy one-year bonds only if their expected holding-period return for two years exceeds the total return on the two-year bond:

$$R_1 + \mathrm{E}(R_1) > 2R_2,$$

which means

$$R_2 < \frac{1}{2}\left[R_1 + \mathrm{E}(R_1)\right].$$

If a bond issuer has a two-year time horizon, then for him issuing one-year bonds is risky (the short-term interest rate might rise the second year) and issuing the two-year bond is safe. If he is risk averse, he will issue the one-year bonds only if the expected holding-period return for two years is less than the total two-year return on the two-year bond:

$$R_1 + \mathrm{E}(R_1) < 2R_2.$$

The situation is reversed with a one-year time horizon.

Upward-Sloping Yield Curve

A widespread point of view is that investors tend to have a short-time horizon and that bond issuers have a long time-horizon.

Then market equilibrium requires

$$R_2 > \frac{1}{2}\left[R_1 + \mathrm{E}\left(R_1\right)\right].$$

Otherwise investors would buy only one-year bonds and bond issuers would issue only two-year bonds, which would be inconsistent with market equilibrium.

If interest rates are expected to stay constant ($R_1 = \mathrm{E}(R_1)$), the yield curve will be upward sloping, $R_2 > R_1$.

One does observe an upward-sloping yield curve more often than a downward-sloping yield curve.