Random Walk

In probability theory, a random walk is a stochastic process in which the change in the random variable is uncorrelated with past changes.

Hence the change in the random variable cannot be forecasted.

For a random walk, there is no pattern to the changes in the random variable, as the existence of any pattern would mean that the changes can be forecasted.

Random Walk for Stock Price

In an efficient market, typically a stock price should approximately follow a random walk.

Otherwise the price change on the stock could be forecasted, and there would be an opportunity for economic profit.

Efficient-Market Theory

The efficient-market theory implies that there should be no pattern to the rate of return on the asset, as the existence of any pattern would mean that the rate of return could be forecasted.

News

Although the change in the stock price cannot be forecasted, the change is not irrational. News about economic fundamentals—sales, earnings, dividends, interest rates, the business cycle, etc.—is what causes the price to change.

Qualification

The rate of return is the expected rate of return plus the unexpected rate of return.

In an efficient market, what cannot be forecasted is the unexpected rate of return. There can be no pattern to the unexpected rate of return.

The expected rate of return is the market interest rate. If the market interest rate is not constant, then an investor can see how it is changing, and in this sense the rate of return can be forecasted somewhat.

Special Circumstances

In special circumstances, market efficiency does not imply that a stock price should follow a random walk.

Ex Dividend Date

Of course the stock price does not follow a random walk at the ex dividend date. In an efficient market, on the ex dividend date the stock price falls by the amount of the dividend. Otherwise there would be an opportunity for economic profit.

Stock price tables in the newspaper take this effect into account. On the ex dividend date, the reported change in the stock price is not the actual price change, but is rather the stock price change adjusted for the dividend. If the stock price falls by exactly the dividend, then the reported price change is zero.

Equal Chance of Price Rise or Fall

Typically one expects approximately an equal chance of a price rise or a price fall.

To have a positive expected rate of return, the expected change in the stock price is *slightly* positive. Hence the probability of a price rise is perhaps *slightly* higher than one half, but is nevertheless very close to one half.

Asymmetry

However in special circumstances these probabilities may differ sharply.

Takeover Offer

Consider a company with share price \$50. Suppose that another company unexpectedly offers to buy the shares for \$80.

Typically the executives of the target company try to fight off the takeover offer, as they are likely to lose their jobs if the takeover is successful.

Suppose that there is a $2/3$ chance that the takeover will succeed, and the share value will be \$80. There is a $1/3$ chance that the takeover will fail, and the share value will fall back to \$50.

In an efficient market, the share price will be \$70: the expected share value is

$$\frac{2}{3} \times 80 + \frac{1}{3} \times 50 = 70.$$

Peso Effect

This same principle can apply generally. Suppose that there is a very small chance of a *big* price fall, but otherwise the price will fluctuate up and down only a small amount.

The possibility of the big price fall lifts the probability of a small price rise. Nevertheless the expected rate of return should be the market interest rate.

Low Risk

For Treasury bills, which have low risk, almost every day the bill price rises.

A price fall can occur only if the unexpected rate of return is more negative than the expected rate of return.

Term Structure

In bond pricing, there is necessarily a pattern to the rate of return. Over the lifetime of the bond, the bond must earn the yield to maturity. A higher rate of return in one year therefore implies a lower rate of return in another.

For long-term bonds, however, the day-to-day or month-to-month price change is approximately a random walk.