

## A Firm in Equilibrium

Consider a firm that owns assets (factories, machinery, etc.) worth  $A$ . Assume that these assets earn profit at the market interest rate  $R$ , so the earnings  $E$  (profit) of the firm is the interest rate times the asset value,

$$E = RA. \quad (1)$$

For simplicity, assume that the firm has no outstanding debt. It uses no external finance, and the number of shares stays constant. The total market value of its stock is  $P$ .

## Market Value Equals Asset Value

If markets are in equilibrium, then it is intuitive that the market value of the stock should equal the asset value,

$$P = A. \tag{2}$$

We show this equality via present value.

## Present Value

In market equilibrium, the market value of a share of stock is the present value of dividends. As the total number of shares is constant, the market value of all the stock is the present value of total dividends. If the current total dividend is  $D$ , and dividends grow at rate  $G$ , then the present value is

$$P = \frac{D}{R - G}, \quad (3)$$

which we showed previously.

## **Dividend Policy**

*Dividend policy* refers to the size of the dividend. The firm may choose to pay either low dividends or high dividends. If dividends are low, then retained earnings (reinvested profits) are high. This high investment will cause the total assets of the firm, and hence earnings, to grow more rapidly. Conversely, high dividends will cause a slower growth of assets and profits.

## Constant Ratio

Assume that the dividend policy is a constant ratio  $D/E$  of dividends to earnings. For a typical large corporation, this ratio is indeed rather constant and is perhaps one half.

## Investment and Growth

The total investment of the firm is the retained earnings, so investment is earnings less dividends,

$$E - D.$$

The growth rate for assets is investment divided by assets,

$$G = \frac{E - D}{A}.$$

Earnings and dividends also grow at this rate, and hence so does the stock price. Higher dividends reduce the growth rate.

## Present Value of Dividends

We calculate the market value of the stock, the present value of dividends, starting with (3):

$$\begin{aligned}
 P &= \frac{D}{R - G} = \frac{D}{R - \left(\frac{E - D}{A}\right)} = \frac{D}{R - \left(\frac{E}{A}\right) \left(1 - \frac{D}{E}\right)} \\
 &= \frac{D}{R - R \left(1 - \frac{D}{E}\right)} = \frac{D}{R \left(\frac{D}{E}\right)} = \frac{E}{R} \\
 &= A,
 \end{aligned}$$

using (1).

## Summary

Thus the market value of the stock indeed does equal the asset value (equation (2)).

Furthermore, the dividend policy has no effect on the market value, even though it does affect the growth rate.

## **No Natural Rate of Capital Gain**

The rate of return on the stock is the dividend yield plus the rate of capital gain. In market equilibrium, the rate of return must be the market interest rate  $R$ .

There is no natural rate of capital gain, as this rate depends on the dividend policy: lowering the dividend reduces the dividend yield and raises the rate of capital gain.

## Stock Buy-Backs

Modify this model by allowing the firm to buy back some of its shares in the marketplace. Let  $B$  denote the total value bought. As shares are bought, the total number of shares outstanding declines, at rate  $B/P$ . Assume that the firm uses a fixed fraction of its earnings to buy back stock, so  $B/E$  is constant.

One can see a stock buy-back as much like a dividend, in that it transfers money from the firm to the stockholders.

## Growth

Reinvested earnings are now  $E - D - B$ , and the growth rate of total assets (and earnings) is

$$\frac{E - D - B}{A}.$$

The growth rate of dividends per share is the growth rate of assets and earnings plus the rate of decline in the number of shares,

$$G = \frac{E - D - B}{A} + \frac{B}{P}.$$

## No Effect on Market Value

Substituting this growth rate into the present value (3), it is possible to solve for  $P$  to obtain (2), that the market value equals the asset value. Like the dividend policy, the stock buy-back has no effect on the current market value.

## **Greater Capital Gain, Less Dividends**

However it does affect the form of the rate of return on the stock. Necessarily the rate of return is the dividend yield plus the rate of capital gain, so replacing dividends with a stock buy-back shifts the rate of return from dividends to capital gains.

## Illegal Tax Dodge

As capital gains are subject to income tax at only half the rate of dividends (and other income), the stock buy-back is a tax dodge for the firm. The Internal Revenue Service has long been aware of this potential tax dodge, and a stock buy-back to avoid tax is illegal.

However in the past decade this law has not been enforced. Dividends have shrunk, and stock buy-backs have soared. The tax revenue loss is tens of billions of dollars per year.