Arbitrage

Arbitrage refers to the simultaneous purchase and sale in different markets to achieve a certain profit.

In market equilibrium, there must be no opportunity for profitable arbitrage.

Otherwise one could make a certain profit by buying low (buying the undervalued asset) and selling high (selling the overvalued asset). There would be excess demand for the former and excess supply for the latter.

Option Pricing

Based on the principle that no arbitrage opportunity can exist, one can develop an elaborate theory of option pricing.

Option-Price Table

Call option prices for Tandy stock:

	April	July	October
40	$3\frac{3}{8}$	$5\frac{5}{8}$	7
45	$\frac{13}{16}$	$2\frac{5}{8}$	$4\frac{5}{8}$
50	$\frac{5}{16}$	1	$2\frac{1}{2}$

The table shows nine option prices, for three striking prices and three expiration dates.

The spot price is $42\frac{7}{8}$. Hence the 40 calls are in-the-money, and the 45 and 50 calls are out-of-the-money.

Patterns

What pricing patterns are present in the table?

Expiration Date

The price rises as the expiration date becomes more distant.

This property conforms with the general principle that expanding one's possible choices has value.

However one can say more: if the price were less for a more distant expiration date, then one could make an arbitrage profit.

Example

Suppose that the 40-July option price was 8.

Since one makes an arbitrage profit by buying low and selling high, one buys the undervalued option and sells the overvalued option.

Hence buy a 40-October option and sell a 40-July option.

One makes an immediate gain of $1 = 8 - 7$. Then what do you do?

Option is Exercised

If the 40-July option is exercised, you immediately exercise the 40-October option. These two transactions cancel, and your overall profit is 1.

Option is Not Exercised

Alternatively, the 40-July option might never be exercised.

You still have the 40-October option, and perhaps you can exercise it for additional profit. Your overall profit is therefore at least 1, and perhaps more.

Striking Price

The price rises as the striking price goes down.

This property conforms with the intuitive principle that to buy for less is better than paying more.

However one can say more: if the price were less for a lower striking price, then one could make an arbitrage profit.

Example

Suppose that the 45-October option price was 8.

Since one makes an arbitrage profit by buying low and selling high, one buys the undervalued option and sells the overvalued option.

Hence buy a 40-October option and sell a 45-October option.

One makes an immediate gain of $1 = 8 - 7$. Then what do you do?

Option is Exercised

If the 45-October option is exercised, you immediately exercise the 40-October option. You are buying a share for 40 and selling a share for 45, so you make a profit of 5. Your total profit is therefore 6.

Option is Not Exercised

Alternatively, the 45-October option might never be exercised.

You still may be able to exercise the 40-October option at a profit. Your overall profit is therefore at least 1, and perhaps more.

No Exercise

It does not pay to exercise an option prior to the expiration date.

Although one could profit by exercising an in-the-money option, one would profit *more* by selling the option.

For example, consider the 40-April option. One could exercise it to get the profit $42\frac{7}{8} - 40 = 2\frac{7}{8}$. However it would be better just to sell the option, for $3\frac{3}{8}$.

The *intrinsic value* of an option is its value if the expiration were immediate: the greater of zero and what one would make by exercising the option now. The claim is that the market value of an option always *exceeds* the intrinsic value.

Notation

S	Stock price
C	Call price
X	Exercise price
R	Risk-free rate of return
T	Time to expiration

No Early Exercise

If one exercises the call at the present time, one has $S - X$.

If one sells the stock short now and covers by exercising the call at expiration, the present value is

$$S - \frac{X}{(1+R)^T},$$

The call must be worth at least this much, so

$$C \geq S - \frac{X}{(1+R)^T} > S - X.$$

Consequently it does not pay to exercise before expiration.

Dividend

This argument assumes that the stock pays no dividend.

If there is a dividend, it may pay to exercise early, to capture the dividend.

Intuition

To exercise an option of course precludes future exercise. The possibility of future exercise has some value, and to exercise an option before expiration forfeits this value. Consequently one should not exercise an option before expiration.