Yield to Maturity

The *yield to maturity* is the rate of return obtained by buying a bond at the *current market price* and holding it to maturity.

No Default

In the calculation of the yield to maturity, one assumes that there will be no default: all payments will be made as promised.

If there is default, then the rate of return actually achieved is less than the yield to maturity.

Coupon Payment

The *coupon payment* refers to the total interest per year on a bond.

The name originated in the past. Long ago coupons were attached to a bond by perforations, somewhat like postage stamps. To receive the interest due, the owner of the bond would clip the next coupon and submit it for redemption to receive the interest.

The phrase *coupon clipper* refers to a wealthy person who lives on bond interest.

Example

Consider a bond with a coupon payment of $80 per year and maturity value $1000 in ten years.

If the current market price is $1000, then what is the yield to maturity?

The yield to maturity must be 8%, since one receives a profit of $80 per year on a $1000 investment.

Lower Bond Price

Alternatively, suppose that the current market price is $900. What is the yield to maturity?

Current Yield

One might say that the yield to maturity is the *current yield*, the coupon payment divided by the current market price,

$$\text{current yield} = \frac{\text{coupon}}{\text{price}} = \frac{80}{900} \approx .089.$$

Understatement

The current yield *understates* the yield to maturity. The investor also receives a $100 profit by holding the bond to maturity, the $1000 maturity value less the $900 purchase price.

One might argue that a \$100 profit is like a \$10 profit per year. Then the total profit each year is the coupon payment plus this gain,

$$80 + 10 = 90.$$

The yield to maturity would then be

$$\frac{90}{900} = .100.$$

Overstatement

This figure *overstates* the yield to maturity. The calculation supposes that a $10 profit is received each year, whereas in fact the total $100 profit is received only at maturity.

A payment of $100 in ten years has a lower present value than $10 received each year for ten years.

Inverse Relation of Price and Yield

A lower current market price raises the yield to maturity, since one receives the same payments for a smaller initial investment.

Asset-Market Equilibrium

The rate-of-return condition and the present-value condition are equivalent conditions for asset-market equilibrium.

The equivalence means that for the price to equal the present value of the payments discounted at R is equivalent to a rate of return equal to R.

Calculating the Yield to Maturity

Hence one calculates the yield to maturity as the discount rate R that makes the current bond price equal to the present value of the payments.

For a bond with current market price P, coupon payment C, and maturity value 1000 after n years, one solves

$$P = \frac{C}{1+R} + \frac{C}{(1+R)^2} + \cdots + \frac{C}{(1+R)^{n-1}} + \frac{C+1000}{(1+R)^n}$$

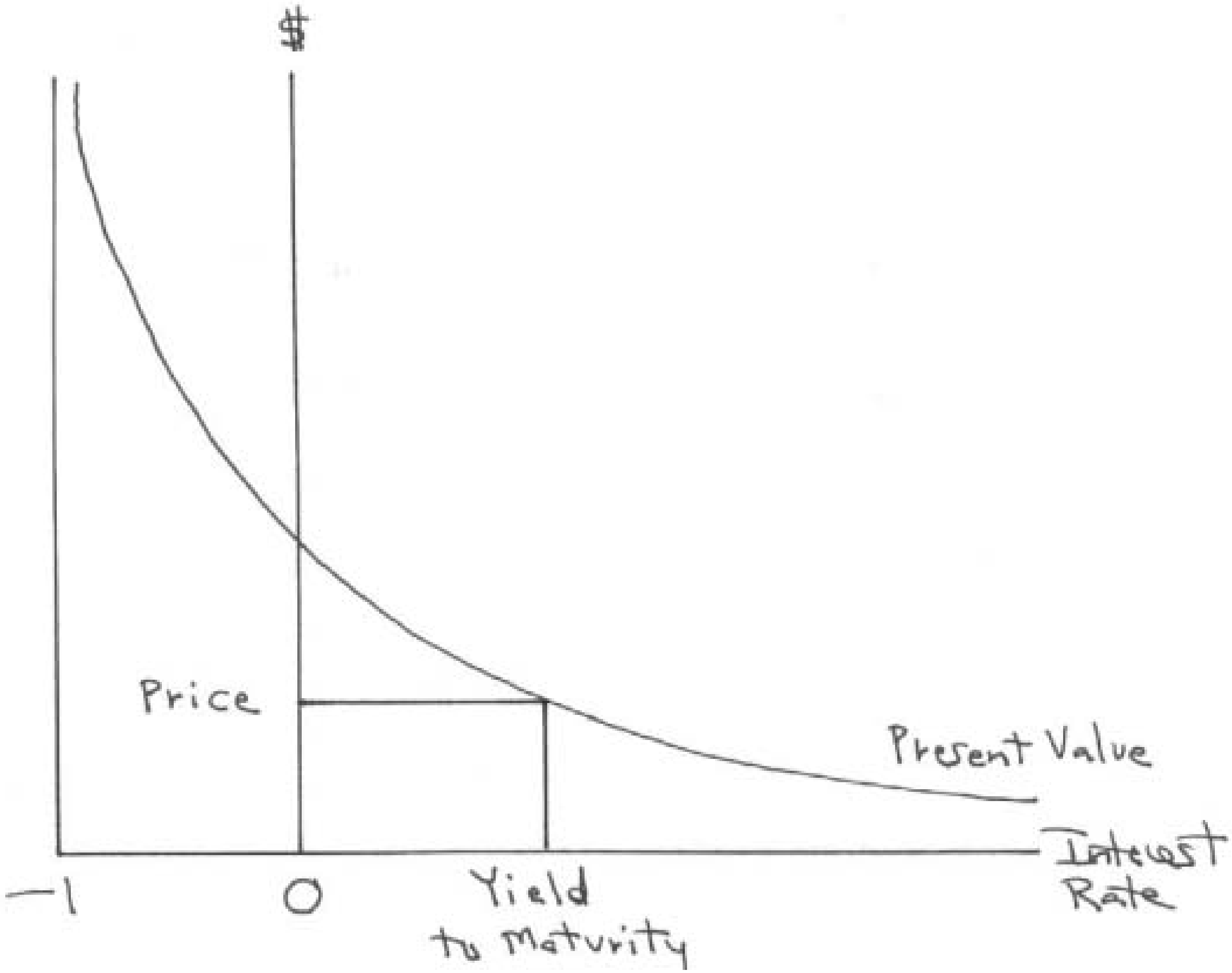
for R.

Inverse Relation of Price and Yield

As the interest rate increases, the present value falls (figure 1). As R goes to infinity, the present value goes to zero. As R goes to minus one, the present value goes to infinity.

Given the price, one finds the yield to maturity as the discount factor that makes the present value equal the price. If the price goes up, then the yield to maturity goes down.

Figure 1: Yield to Maturity



If the price $P = 1000$, then the yield to maturity is

$$R = \frac{C}{1000}.$$