

### Real Theory of Interest

First presented by Irving Fisher [1], the *real theory of interest* argues that real economic variables determine the real interest rate.

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### General Equilibrium

The analytical framework is general equilibrium microeconomic theory. Demand equals supply in every market throughout the economy. The allocation of resources is efficient. No resources are wasted, and no inputs are unemployed.

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### Real Factors Determine Real Variables

Real economic factors determine the real economic variables. Together technology, the endowments of inputs—capital, labor, natural resources—and consumer preferences determine real quantities and relative prices.

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### No Role for Money and Monetary Policy

According to this theory, money and monetary policy have no effect on the real interest rate.

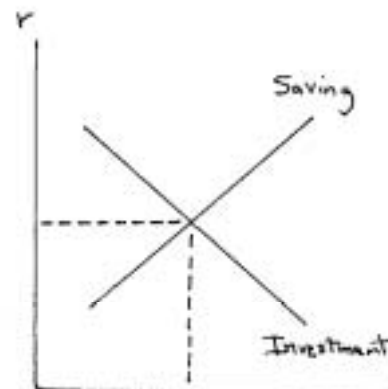
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### Desired Saving Equals Desired Investment

The theory says that the real interest rate  $r$  adjusts so desired saving  $S$  equals desired investment  $I$  (figure 1). As the real interest rate is the cost of capital to the firm, a lower real interest causes higher investment demand. And as the real interest rate is the return to saving, a higher real interest rate creates a greater incentive to save; and saving increases. In equilibrium desired saving equals desired investment.

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Figure 1: Desired Saving and Investment



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However we would like to go beyond this formulation, to explain better how the interest rate affects saving and investment.

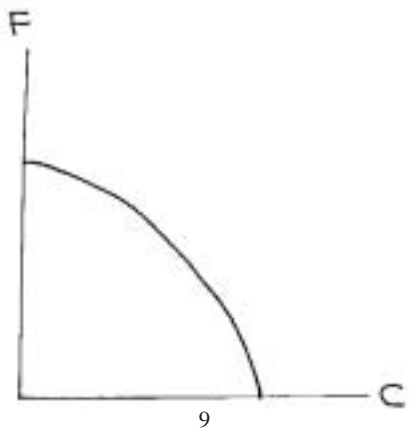
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## The Production Possibility Frontier

The *production possibility frontier* shows the possible combinations of goods that can be produced. For example, consider an economy producing clothing  $C$  and food  $F$  (figure 2). The frontier shows the maximum quantities that can be produced. A point down and right on the frontier represents a concentration on clothing at the expense of food. Conversely, a point up and left on the frontier represents a concentration on food.

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Figure 2: Production Possibility Frontier



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## Efficient Production

A point on the frontier constitutes *efficient* production, as the economy cannot produce more of one good without producing less of the other. In contrast, a point inside the frontier is inefficient: available inputs are not being utilized efficiently, and the economy could produce more of both goods.

10

## Slope is Tradeoff

The slope of the frontier shows the technological *tradeoff* of food versus clothing. For example, if the slope is  $-2$ , then the tradeoff is left one and up two—one unit of clothing for two units of food. The slope becomes steeper down and right along the frontier. The convex shape for the frontier stems from diminishing marginal returns. As inputs are shifted from producing one good to producing the other, diminishing marginal returns set in. The good now produced in a greater quantity becomes more costly (higher tradeoff) relative to the other good.

11

## The Intertemporal Production Possibility Frontier

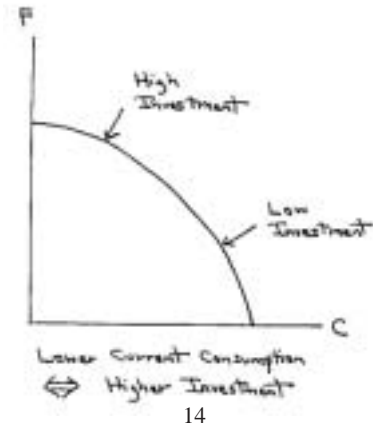
We reinterpret figure 2 as an *intertemporal* production possibility frontier. Now the horizontal axis is current consumption, and the vertical axis is future consumption. The frontier shows the efficient combinations of current and future consumption that the economy can produce.

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### Investment

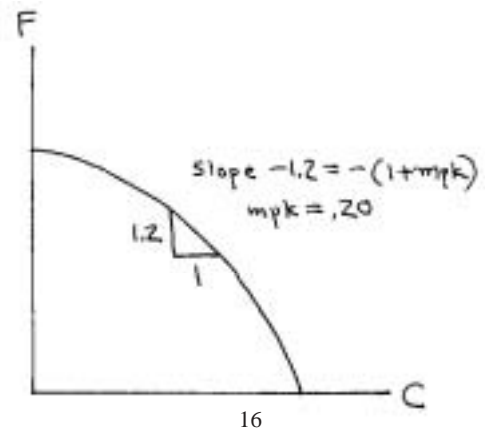
Underlying the different points on the frontier are different levels of investment (figure 3). For example, a point down and right on the frontier has high current consumption and low future consumption. The high current consumption means that current investment must be low. As this investment is low, in the future the capital stock is low, and consequently the production of future consumption is low. Conversely, a point up and left on the frontier has high investment. This high investment comes at the expense of current consumption and makes possible high future consumption.

Figure 3: Investment



The slope of the frontier shows the tradeoff of current consumption for future consumption. For example, slope  $-1.20$  signifies that to reduce the production of current consumption by one allows future consumption to increase by 1.20.

Figure 4: Marginal Product of Capital



### Marginal Product of Capital

We define the *marginal product of capital* via this slope (figure 4). For this example, the marginal product of capital  $mpk = .20$ . (The marginal product of capital is the absolute value of the slope, less one.) One can see this value as a sort of return to investment. Reducing current consumption by one allows investment to increase, which then raises future consumption by one plus the marginal product of capital.

### One-Sector Technology

Economists often think in terms of a *one-sector technology*, in which there is only one produced good. Consumption and capital are the same good. National product is divided into consumption and investment. To reduce consumption by one unit raises investment by one unit. Even though the economy actually produces millions of distinct goods, this simplification is useful.

The terminology “marginal product of capital” stems from the concept of a one-sector technology. Then a marginal product of capital  $mpk = .20$  means that to reduce current consumption by one raises investment by one and the current capital stock rises by one. This extra capital raises the national product next year by its marginal product,  $mpk = .20$ . In the future, consumption rises by  $1.20$ , as this extra product is consumed and in addition the extra unit of capital is consumed.

19

## Supply

*Supply* refers to the quantity of goods produced by the firms in the economy. Microeconomic theory shows how profit maximization leads firms to produce on the production possibility frontier. To produce inside the frontier would mean that profit is not maximized, as more could be produced. In particular, firms produce at the point on the frontier where the absolute value of the slope is the relative price of the two goods.

20

This situation applies to the intertemporal production possibility frontier. For example, suppose that the real interest rate is  $r = .10$ . By saving one unit of current income, one earns interest of  $.10$ . In the future one can consume this saving plus the interest earned. Consequently one effectively trades one unit of current consumption for  $1 + r = 1.10$  units of future consumption. The relative price of current and future consumption is  $1 + r$ .

21

## Profit Maximization

To maximize profit, a firm invests so that the return to investment equals the cost of investment. If the return were higher than the cost, for example, then the firm would raise its profit by increasing investment. The return to investment is the marginal product of capital  $mpk$ , and the cost of capital is the real interest rate  $r$ . Profit maximization requires that

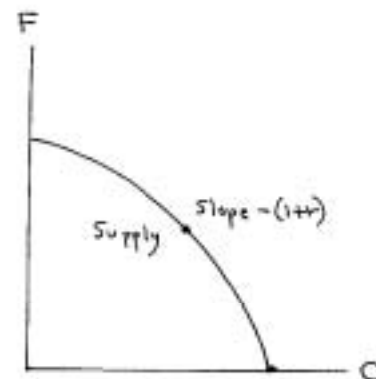
$$mpk = r.$$

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Equivalently, as the absolute value of the slope of the intertemporal production possibility frontier is  $1 + mpk$ , profit maximization requires that the absolute value of the slope must be  $1 + r$ .

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Figure 5: Supply



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The real interest rate therefore determines the supply of current and future consumption (figure 5). If the real interest rate is high, firms produce where the frontier is steep—high current consumption, low future consumption, with low investment. If the interest rate is low, firms produce where the frontier is relatively flat—low current consumption, high future consumption, with high investment. Thus a fall in the real interest rate raises investment.

25

## Demand

Consider a consumer having current income  $Y_C$  and future income  $Y_F$  and having no other resources. He finances his current and future consumption entirely from this income. One choice for him is just to consume his income, both now and in the future:  $C = Y_C$  and  $F = Y_F$ .

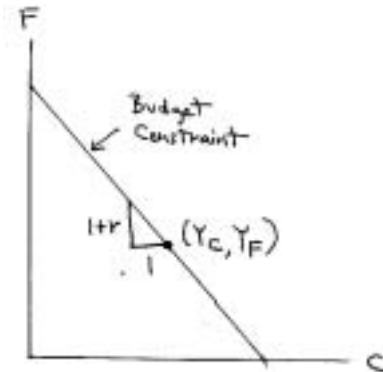
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## Budget Constraint

In addition, suppose that the consumer can borrow or lend at the real interest rate  $r$ . The opportunity to borrow or to lend expands his possible choices of current and future consumption. The *budget constraint* is the possible combinations  $(C, F)$  that are possible (figure 6).

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Figure 6: Budget Constraint



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For example, the consumer might decide to save one real unit, which would reduce current consumption to  $C = Y_C - 1$ . The interest earned on the extra saving would cause future consumption to rise to  $F = Y_F + (1 + r)$ . Alternatively, if instead the consumer saves two real units, then  $C = Y_C - 2$  and  $F = Y_F + 2(1 + r)$ .

29

Going the other direction, the consumer might borrow. If he borrows one real unit and repays the loan from his future income, then his current consumption rises by one,  $C = Y_C + 1$ , and his future consumption falls to  $F = Y_F - (1 + r)$ .

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Figure 6 shows the possibilities: the budget constraint is just the straight line through  $(Y_C, Y_F)$  with slope  $-(1+r)$ . The relative price of current and future consumption sets the slope. A higher real interest rate means that future consumption is less expensive relative to current consumption, and the slope of the budget constraint is steeper.

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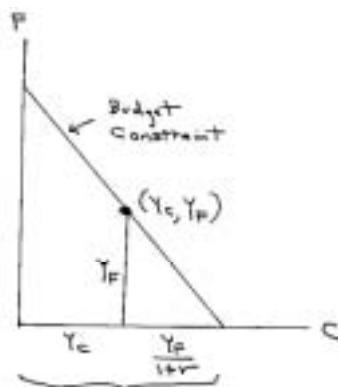
## Present Value

One can relate the budget constraint to present value (figure 7). The horizontal intercept is the present value of income  $Y_C + \frac{Y_F}{1+r}$ . The budget constraint is that the present value of consumption equals the present value of income,

$$C + \frac{F}{1+r} = Y_C + \frac{Y_F}{1+r}.$$

32

Figure 7: Present Value of Income



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## Demand

The *demand* is the point on the budget constraint that the consumer chooses, the point that he regards as most preferred. Whether he chooses to borrow or to lend depends on his preference for current versus future consumption.

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## Aggregate Budget Constraint

Combining the budget constraint for each consumer gives the economy-wide *aggregate budget constraint*, the budget line for all consumers together. The total demand for all consumers is the point on the aggregate budget line resulting from the choices of each consumer.

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## Supply and Demand

The national income equals national product identity implies that the production of goods necessarily creates exactly the purchasing power needed to buy the goods.

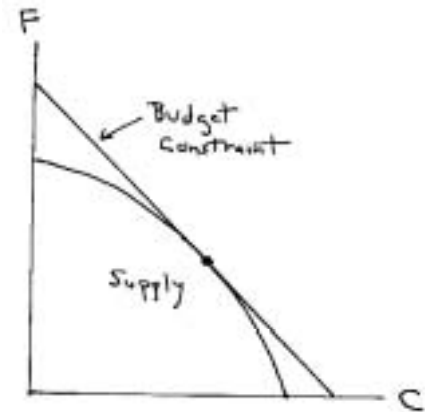
The same principle applies in an intertemporal setting. The supply—the production of current and future consumption—creates exactly the purchasing power needed to buy the consumption. Consequently the supply is one point on the aggregate budget constraint.

36

### Budget Constraint as Tangent Line

At the supply point, the slope of the intertemporal production possibility frontier is  $-(1+r)$ . As this value is also the slope of the budget line, the aggregate budget constraint is just the line tangent to the frontier at the supply (figure 8). Demand is then the point on this tangent line chosen by the consumers.

Figure 8: Budget Constraint as Tangent Line



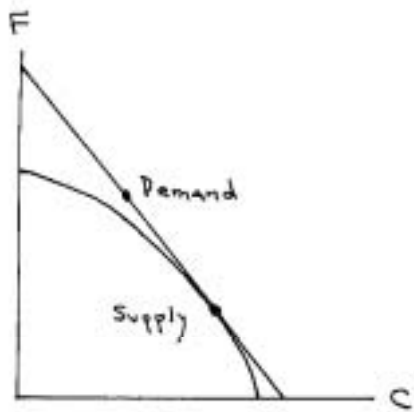
### Market Equilibrium

For an appropriate real interest rate  $r$ , the demand equals the supply, and there is *market equilibrium*. One can show that at market equilibrium desired saving equals desired investment.

### High Real Interest Rate

Consider a high interest rate (figure 9). The supply is down and right on the frontier. With a high interest rate, consumers have a strong incentive to save, so one expects that the demand will be up and left from the supply, along the budget constraint. Desired saving exceeds desired investment, and demand does not equal supply.

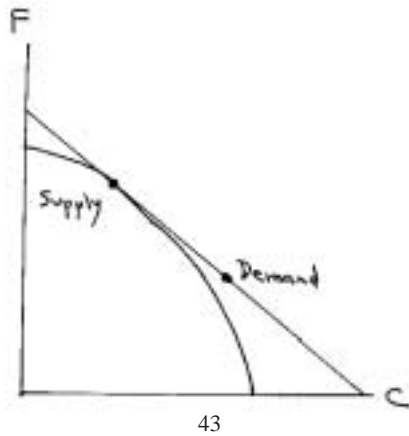
Figure 9: High Real Interest Rate



### Low Real Interest Rate

A low real interest rate is the opposite case (figure 10). The supply is up and left on the frontier. With a low interest rate, consumers have no incentive to save, so one expects that the demand will be down and right from the supply. Now desired saving is less than desired investment, and demand does not equal supply.

Figure 10: Low Real Interest Rate



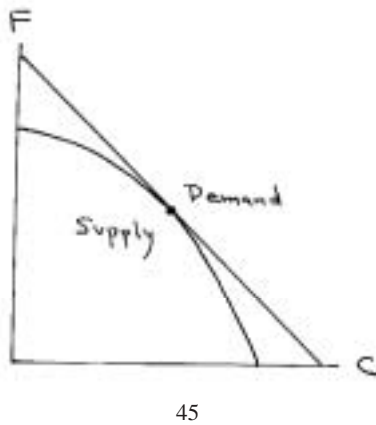
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### Equilibrium

Equilibrium occurs only if the real interest rate is appropriate (figure 11). Then the demand and the supply coincide, and desired saving equals desired investment. Overall the equilibrium real interest rate depends on both supply and demand.

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Figure 11: Equilibrium



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Traditionally, one summarizes the theory by the terminology that the productivity of capital and time preference together determine the real interest rate. The intertemporal production possibility frontier represents “productivity of capital.” And time preference determines demand, as the preference for current versus future consumption sets the demand along the budget constraint.

46

### Flat Production Possibility Frontier

A common point of view in economics is the extreme case that the production possibility frontier is flat, not curved. The relative cost of production determines relative prices, and the demand determines the mix of goods produced.

For example, suppose that the technological tradeoff between clothing and food is two for one. Then the price of food will necessarily be twice the price clothing, regardless of demand. The amounts of clothing and food produced are then set by the demand for these goods.

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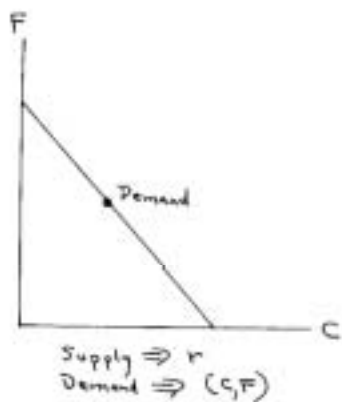
### Flat Intertemporal Production Possibility Frontier

An analogous point of view exists for the intertemporal production possibility frontier (figure 12). Perhaps the frontier is flat—the marginal product of capital is constant.

48



Figure 12: Flat Frontier



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Then the slope of the frontier sets the real interest rate, regardless of the preferences of consumers for current versus future consumption.

The aggregate budget constraint coincides with the frontier, and consumer preferences then set the quantities of current and future consumption along the frontier.

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## References

- [1] Irving Fisher. *The Theory of Interest as Determined by Impatience To Spend Income and Opportunity To Invest It*. Reprints of Economic Classics. A. M. Kelley, New York, 1965. (Originally published 1907) HB539F54 1965.

51