

### Portfolio Balance

The portfolio-balance model of Tobin [1] provides a monetary theory of the interest rate. One models the portfolio demand for financial assets, and the interest rate adjusts to equilibrate the supply and the demand for financial assets.

1

### Real Theory of Interest

In the real theory of interest, the productivity of capital and the consumption/saving choice by households are the key determinants of the interest rate. Together investment and saving interact to set the interest rate.

In contrast, these factors play no role in the portfolio balance model and are completely absent from the model.

2

### Monetary Theory of Interest

Instead, the portfolio-balance model furnishes a monetary theory of interest. A change in monetary policy affects the interest rate. Increasing the money supply makes the interest rate lower.

In the real theory of interest, in contrast, money and monetary policy affect only the overall price level in the economy and have no effect on the real interest rate.

3

### Portfolio Balance

For simplicity, assume that there are only two financial assets—money and bonds. *Money* is the medium of exchange and pays no interest. *Bonds* refers to all assets except money and thus refers to equity as well as debt. Bonds pay the nominal interest rate  $R$ .

The portfolio balance model revolves around the choice of whether to hold wealth as money or as bonds.

4

### Financial Market Equilibrium

In financial market equilibrium, supply equals demand for money and bonds,

$$m^s = m^d \quad (1)$$

$$b^s = b^d. \quad (2)$$

Equilibrium consists of a balance between portfolio supply and demand.

5

### Stocks, Not Flows

Here money and bonds are stock variables, not flow variables. The supply of money and bonds is simply the outstanding stock of money and bonds in the economy. The supply is exogenous (taken as given, determined outside the model).

Define (financial) wealth  $w$  as the total supply of financial assets (money plus bonds), so

$$w \equiv m^s + b^s. \quad (3)$$

Thus wealth is also exogenous.

6

### Portfolio Budget Constraint

The demand for money and bonds is a portfolio demand, showing the allocation of wealth between money and bonds. Given wealth, people decide what amount to hold as money and as bonds. A dollar of wealth must be held either as money or as bonds. The *budget constraint* is that wealth must equal the total demand for money and bonds,

$$w = m^d + b^d. \quad (4)$$

7

### Demand and Supply

The demand for money and bonds is endogenous, depending on economic variables determined within the model. The demands are the net demands of the private sector. For example, households hold some bonds, and firms have an outstanding issue of bonds to finance investment. The net demand is the demand by households less the supply by firms.

The money supply is the amount of money supplied by the government. The bond supply is the total capital stock in the economy plus the government debt.

8

### Real Economic Variables

With the exception of the nominal interest rate  $R$ , all variables are real. For example, the real money supply is the nominal money supply divided by price level,

$$m^s = \frac{M^s}{P}.$$

The real supply of bonds is the nominal supply divided by the price level,

$$b^s = \frac{B^s}{P}.$$

When analyzing the economy via the portfolio balance model, it is common to take the price level  $P$  as exogenous.

9

### Comparison with Saving and Investment

One must not confuse saving and investment with portfolio supply and demand. Whereas saving and investment are flow variables, the portfolio supply and demand are stock variables. At most, saving and investment may cause a gradual change in portfolio supply. Portfolio demand deals just with the choice of whether to hold money or bonds and is unaffected by changes in saving and investment.

10

### Portfolio Demands

We suppose that the portfolio demands depend on the real national income and product  $y$ , the nominal interest rate  $R$ , and real wealth  $w$ :

$$m^d(y, R, w)$$

$$b^d(y, R, w).$$

11

### Inventory Model

To understand the portfolio demands, think of Baumol's inventory model of the demand for money. In Baumol's model, an individual chooses his optimum money demand from the trade-off between foregone interest and the cost of trips to the bank, to obtain money. When the interest rate is high, the individual keeps his average money demand low, to reduce the interest foregone. When the interest rate is low, he lets his money demand rise, to reduce the cost of his trips to the bank.

12

### Income Effect

Holding the other variables constant ( $R$  and  $w$  constant), an increase in the national income and product  $y$  raises the demand for money, as the economy requires more money to carry out more purchases and sales. With wealth constant, an increase in demand for money means a decrease in the demand for bonds,

$$y \uparrow \Rightarrow m^d \uparrow \\ \Rightarrow b^d \downarrow,$$

by the budget constraint (4). To carry out increased transactions, money is substituted for bonds.

13

### Interest-Rate Effect

Holding other variables constant ( $y$  and  $w$  constant), an increase in the nominal interest rate  $R$  reduces the demand for money. When the interest rate is higher, people economize on money, to increase the portfolio demand for bonds, to earn the higher interest.

$$R \uparrow \Rightarrow m^d \downarrow \\ \Rightarrow b^d \uparrow.$$

By the budget constraint (4), a higher demand for bonds necessitates a lower demand for money.

14

### The Nominal Interest Rate: The Real Cost of Holding Real Money Balances

Since the other variables are real, it might seem odd that the *nominal* interest rate  $R$  is what affects the demand for money. However the *nominal* interest measures the *real* cost of holding *real* money balances.

15

One unit of real money balances is  $P$  dollars, as  $P/P = 1$ , so the nominal interest foregone by holding one unit of real balances is  $RP$ . The real cost is the nominal interest divided by the price level,

$$\frac{RP}{P} = R.$$

Thus the *real* cost of holding *real* money balances is the *nominal* interest rate.

16

Another way to understand this result is to realize that the interest on bonds less the interest on money is  $R - 0 = R$ ; this comparison is unaltered by the rate of inflation. As the nominal interest rate determines the attractiveness of bonds compared to money, the nominal interest rate is what counts for the portfolio demands.

17

### Wealth Effect

Holding other variables constant ( $y$  and  $R$  constant), one might expect that an increase in wealth would raise both the demand for money and the demand for bonds,

$$w \uparrow \Rightarrow m^d \uparrow \text{ and } b^d \uparrow.$$

If wealth increases by one, then the sum of the increase in the demand for money and the demand for bonds must be one, to satisfy the budget constraint (4).

18

### Walras's Law

The key idea in the portfolio-balance theory is that the interest rate adjusts to achieve the market equilibrium. A change in the interest rate sets the demand for money and bonds into balance with the supply of money and bonds.

19

### Two Equations, One Unknown

In the market equilibrium equations (1)-(2), it might seem that perhaps no interest rate would bring about equilibrium. We have two equations but only one interest rate, so one might think that there is no solution to the equations.

If the interest rate adjusts to make money demand equal to money supply, then perhaps there is disequilibrium for bonds. And if the interest rate adjusts to make bond demand equal to bond supply, then perhaps there is disequilibrium for money.

20

### Only One Independent Equation

However this way of thinking is naive and incorrect: there is really only one independent condition for market equilibrium. The two equations for market equilibrium are actually the same, so adjustment of the interest rate is sufficient to bring about equilibrium for both money and bonds.

21

### Example

Consider an example. Suppose that money supply is 1000, and bond supply is 9000,

$$m^s = 1000$$

$$b^s = 9000.$$

Wealth is therefore

$$w = m^s + b^s = 1000 + 9000 = 10000.$$

22

Suppose that money demand is

$$\begin{aligned} m^d &= 10y - 5R + .2w \\ &= 10y - 5R + 2000. \end{aligned}$$

An increase in  $y$  raises the demand for money, an increase in  $R$  reduces the demand for money, and an increase in  $w$  raises the demand for money.

23

By the budget constraint (4), the demand for money sets the demand for bonds,

$$\begin{aligned} b^d &= w - m^d = w - (10y - 5R + .2w) = -10y + 5R + .8w \\ &= -10y + 5R + 8000. \end{aligned}$$

An increase in  $y$  lowers the demand for bonds, an increase in  $R$  raises the demand for bonds, and an increase in  $w$  raises the demand for bonds.

24

### Market Equilibrium

Here the equations (1)-(2) for market equilibrium are

$$1000 = 10y - 5R + 2000 \quad (5)$$

$$9000 = -10y + 5R + 8000. \quad (6)$$

However the second equation is just a rearrangement of the first, so adjustment of the interest rate is indeed sufficient to bring about equilibrium.

25

### Walras's Law

One can also see the equivalence of the two equilibrium equations by a formal argument. Walras's law asserts that the sum of the excess demand for money and the excess demand for bonds is always zero,

$$(m^d - m^s) + (b^d - b^s) = 0, \quad (7)$$

regardless of  $m^s$ ,  $b^s$ ,  $y$ , and  $R$ .

26

The proof is simple:

$$\begin{aligned} (m^d - m^s) + (b^d - b^s) &= (m^d + b^d) - (m^s + b^s) \\ &= w - w \\ &= 0. \end{aligned}$$

The first equality is a rearrangement. The second equality follows from the budget constraint (4) and the definition of wealth (3).

27

Walras's law implies that the two equations (1)-(2) for market equilibrium are equivalent. Suppose that the first equation holds, that money supply equals money demand. By Walras's law,  $m^d - m^s = 0$  implies  $b^d - b^s = 0$ , so bond supply equals bond demand.

Consequently, one can analyze market equilibrium via either money supply equals money demand, or, equivalently, bond supply equals bond demand.

28

### The LM Curve

One refers to the market equilibrium equations (1)-(2) as the *structural equations*; they show the structure of the financial sector. Given the exogenous variables  $m^s$ ,  $b^s$ , and  $y$ , one solves for the endogenous variable  $R$ . The *reduced form* shows this relationship. In Keynesian macroeconomics, one refers to the reduced form as the *LM curve*.

29

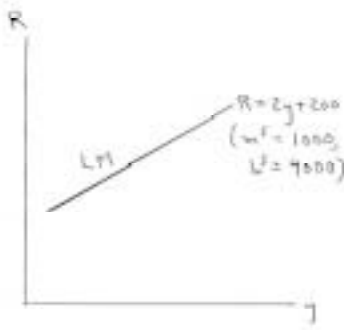
For example, solving either of the structural equations (5)-(6) gives the reduced form

$$R = 2y + 200. \quad (8)$$

Given the supply of money and bonds, the LM curve shows how a change in the national income and product affects the interest rate.

30

Figure 1: LM Curve



### Structural Equations and Reduced Form

The structural equations and the reduced form are equivalent. From the structural equations, one solves for the reduced form. Conversely, from the reduced form, one can solve for the structural equations.

Consider our example again, and write the structural equations as

$$m^s = 10y - 5R + .2w = 10y - 5R + .2(m^s + b^s)$$

$$b^s = w - m^d = w - (10y - 5R + .2w) \\ = -10y + 5R + .8(m^s + b^s).$$

Solve either structural equation to obtain the reduced form

$$R = 2y - .16m^s + .04b^s. \tag{9}$$

One obtains (8) when  $m^s = 1000$  and  $b^s = 9000$ .

One can work backwards from the reduced form (9) to obtain the structural equations. Using the definition of wealth (3), rewrite the reduced form as

$$R = 2y - .16m^s + .04(w - m^s).$$

Solving for  $m^s$  yields

$$m^s = 10y - 5R + .2w.$$

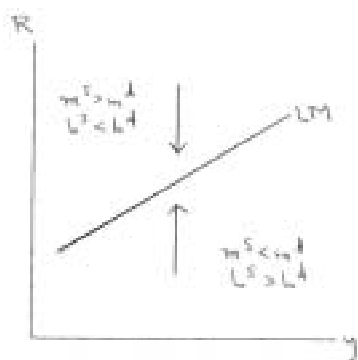
This equation is the first structural equation, and the right-hand side is the demand for money. Alternatively, solving the reduced form for  $b^s$  yields the second structural equation.

### Adjustment to Market Equilibrium

The economic forces of demand and supply push the interest rate  $R$  toward market equilibrium. Consider the LM curve. Each point  $(R, y)$  in the graph represents a possible state for the economy. Along the curve, there is market equilibrium for money and bonds.

Below the curve there is excess demand for money ( $m^s < m^d$ ) and excess supply of bonds ( $b^s > b^d$ ). To understand why there is excess demand for money, consider some point on the LM curve, and then increase  $y$ . The increase raises the demand for money, so now there must be excess demand of money. By Walras's law, excess demand for money implies excess supply of bonds. Above the curve, the opposite condition prevails.

Figure 2: Adjustment to Market Equilibrium



Suppose that the economy lies below the LM curve, so the market is not in equilibrium. Since there is excess supply of bonds, the price of bonds falls. A lower bond price is equivalent to a higher interest rate, so the interest rate rises. Thus market forces do push the interest rate up toward equilibrium.

### Upward Sloping LM Curve

The LM curve is upward sloping: given the money supply and the bond supply, an increase in the national income and product raises the interest rate. We see this property in the reduced form (8) and (9): as  $y$  rises,  $R$  rises.

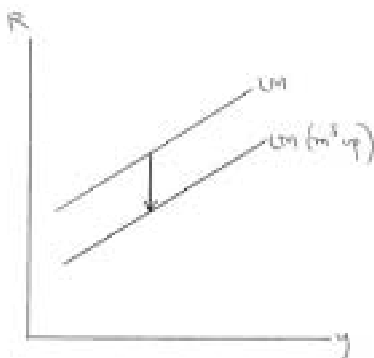
That the LM curve is upward sloping is a general result. Consider an economy initially in market equilibrium, along the LM curve. Suppose that national income and product rises. As explained in the previous section, there is now excess demand for money and excess supply of bonds. The excess supply of bonds forces the bond price down and hence the interest rate up. Thus the new equilibrium interest rate must be higher.

### Monetary Policy

In the portfolio balance model, money and monetary policy are not neutral: an increase in the money supply causes the interest rate to fall.

This property is clear for our example. In the reduced form (9), the coefficient of  $m^s$  is negative—a higher money supply lowers the interest rate. In addition, an increase in the bond supply raises the interest rate. When the bond supply rises, bonds must offer a higher interest rate, to induce a portfolio demand shift toward bonds.

Figure 3: Increase in Money Supply



### Money and Interest

The inverse relation between the money supply and the interest rate is a general result. Consider an economy initially in market equilibrium, along the LM curve. Suppose that the money supply rises. There is now excess supply of money; by Walras's law, there is excess demand for bonds. This excess demand forces upward the price of bonds; equivalently, the interest rate falls. Thus the new equilibrium interest rate is lower than previously. The increase in the money supply causes the LM curve to shift down.

## References

- [1] James Tobin. A general equilibrium approach to monetary theory. *Journal of Money, Credit and Banking*, I(1):15–29, February 1969. HG201J6.