Hyperinflation

Hyperinflation refers to very rapid inflation. For example, prices may double each month. If prices double each month for one year, the price level increases by the factor $2^{12} = 4,096$, so money loses almost all its value.

The cause of hyperinflation is rapid growth in the money supply.

The most famous hyperinflation was in Germany, from 1921-1923 [3].

Opportunity Cost of Holding Money

In Baumol's inventory model of the demand for money [1], the opportunity cost of holding money is the nominal interest rate, measuring the interest foregone by holding money. As the interest rate rises, the real money demand falls.

Under hyperinflation, Cagan [2] argues that opportunity cost of holding money is best measured by the rate of inflation. High inflation causes money to lose its purchasing power rapidly. As the rate of inflation rises, the real money demand falls.

Cagan's Demand for Money in Hyperinflation

The nominal money supply is M_t, and the price level is P_t. Let Π_t denote the rate of inflation at time t, and let $\mathrm{E}\left(\Pi_t\right)$ denote the expected rate of inflation at time t.

In Cagan's model, the real demand for money is inversely related to the expected rate of inflation:

$$\frac{M_t}{P_t} = -a\mathrm{E}\left(\Pi_t\right). \tag{1}$$

The coefficient $a > 0$ measures the sensitivity of the demand for money to expected inflation.

Adaptive Expectations

Adaptive expectations means that expected inflation adjusts gradually in response to actual inflation:

$$\mathrm{E}(\Pi_t) = \mathrm{E}(\Pi_{t-1}) + b\left[\Pi_{t-1} - \mathrm{E}(\Pi_{t-1})\right] \tag{2}$$

$$= b\Pi_{t-1} + (1-b)\,\mathrm{E}(\Pi_{t-1}). \tag{3}$$

in which b lies between zero and one. Expected inflation increases or decreases depending on whether actual inflation is greater or less than expected inflation. If b is near zero, then expected inflation adapts slowly to current inflation; if b is near one, then expected inflation adapts quickly.

Geometric Distributed Lag

Under adaptive expectations, expected inflation is a geometric distributed lag function of past inflation.

Repeated substitution of equation (3) yields the following.

$$
\begin{aligned}
\mathrm{E}\left(\Pi_t\right) &= b\Pi_{t-1} + (1-b)\,\mathrm{E}\left(\Pi_{t-1}\right) \\
&= b\Pi_{t-1} + (1-b)\left[b\Pi_{t-2} + (1-b)\,\mathrm{E}\left(\Pi_{t-2}\right)\right] \\
&= b\Pi_{t-1} + b(1-b)\,\Pi_{t-2} + (1-b)^2\,\mathrm{E}\left(\Pi_{t-2}\right) \\
&= b\Pi_{t-1} + b(1-b)\,\Pi_{t-2} \\
&+ (1-b)^2\left[b\Pi_{t-3} + (1-b)\,\mathrm{E}\left(\Pi_{t-3}\right)\right] \\
&= b\Pi_{t-1} + b(1-b)\,\Pi_{t-2} + b(1-b)^2\,\Pi_{t-3} \\
&+ (1-b)^3\,\mathrm{E}\left(\Pi_{t-3}\right) \\
&= b\Pi_{t-1} + b(1-b)\,\Pi_{t-2} + b(1-b)^2\,\Pi_{t-3} + \cdots .
\end{aligned}
$$

Weighted Average

In the final expression, expected inflation is a weighted average of past inflation. The weights on the lagged inflation add to one,

$$b + b(1-b) + b(1-b)^2 + \cdots = \frac{b}{1-(1-b)} = 1,$$

using the formula for an infinite geometric sum.

The most recent inflation receives the highest weight, and the weight declines for lagged inflation. If inflation stays constant, then expected inflation equals this constant value.

Econometrics

Cagan finds that this model fits quite well for the German hyperinflation. Choosing a value for the adaptive parameter b determines expected inflation. One picks a and b so that the model (1) explains real money balances as closely as possible.

As inflation increases, expected inflation increases, and the demand for real money balances falls.

Cagan concludes that the demand for money seems to be a stable function of observable variables, even in the extreme circumstance of hyperinflation.

Non-Neutrality of Money

In the Cagan model, money is *not* neutral. Increasing the growth rate of money raises inflation and reduces the demand for real money balances.

Printing Money to Finance the Deficit

During the hyperinflation, suppose that the German government was financing a fixed real deficit by printing money.

Printing money ΔM_t finances real expenditure

$$\frac{\Delta M_t}{P_t}.$$

(The first-differences ΔM_t and ΔP_t are the changes in the variables.)

Multiplying and dividing by M_t gives

$$\frac{\Delta M_t}{P_t} = \left(\frac{\Delta M_t}{M_t}\right)\left(\frac{M_t}{P_t}\right),$$

which expresses the real expenditure financed as money growth times real money balances. As the hyperinflation progressed and real money demand M_t/P_t fell, to cover the same real government deficit the growth rate $\Delta M_t/M_t$ of money must *increase*, which causes inflation $\Delta P_t/P_t$ to *increase*.

This prediction is consistent with the facts: as the hyperinflation progressed, the German government did increase the growth rate of money, pushing the rate of inflation even higher.

References

[1] William J. Baumol. The transactions demand for cash: An inventory theoretic approach. *Quarterly Journal of Economics*, LXVI(4):545–556, November 1952. HB1Q3.

[2] Phillip Cagan. The monetary dynamics of hyperinflation. In Milton Friedman, editor, *Studies in the Quantity Theory of Money*, pages 25–117, Chicago, 1956. University of Chicago Press. HG221F87.

[3] *The Economist*. Loads of money, December 31 1999. HG11E2.