Transactions Demand for Money

Money is the medium of exchange, and people hold money to make purchases.

Economists speak of the *transactions demand* for money, as people demand money to make transactions.

Inventory Model of the Demand for Money

Baumol [1] presents an inventory model of the demand for money.

A firm holds inventory so customers can buy. When the inventory is depleted, the firm replenishes the inventory.

In the same way, an individual holds an inventory of money, to use for purchases. When the inventory is depleted, the individual replenishes the inventory.

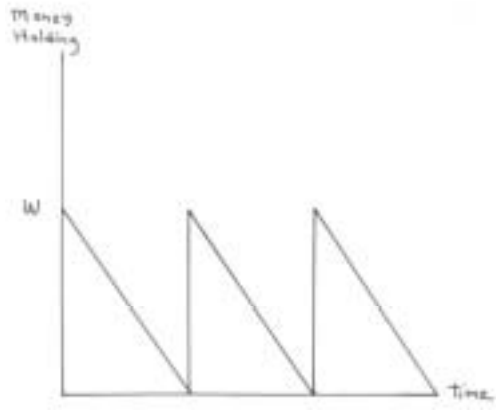
Inventory Management

Consider an individual whose paycheck is automatically deposited in a savings account at the bank, and the deposit earns the interest rate R.

The individual periodically goes to the bank, withdraws W dollars, gradually spends this money, and then returns to the bank when he runs out of money.

The money holding by the individual follows a sawtooth pattern (figure 1).

Figure 1: Money Holding



Spending

Suppose that the individual spends T dollars per year. For example, if $T = 3650$, then the individual spends 10 per day.

If $W = 70$, then the individual would make one withdrawal per week, 52 per year.

The total number of trips to the bank per year is

$$\frac{T}{W}.$$

Cost of Trips to the Bank

Let C denote the cost of a trip to the bank. In particular, this cost would include the value to the individual of the time taken.

The total cost per year of the trips to the bank is the cost per trip multiplied by the number of trips,

$$\frac{CT}{W}.$$

One Trip Per Year?

At one extreme, the individual might choose to make only one trip to the bank per year, by withdrawing $W = T$ dollars at the beginning of the year. Throughout the year the individual would gradually spend this money.

Of course this action would minimize the total cost of the trips to the bank.

Loss

By making only a single trip to the bank per year, the individual would lose interest. The dollars in the wallet potentially could have been on deposit in the bank, earning interest.

Hence it may be better to make a smaller withdrawal but more trips to the bank.

Foregone Interest

Since each withdrawal is W dollars, on average the individual holds $W/2$ dollars in his wallet (figure 1).

The interest foregone per year is the interest rate multiplied this average money holding,

$$\frac{RW}{2}.$$

One Trip Per Day?

At the opposite extreme, the individual might go the bank daily, to keep as much money as possible on deposit, to maximize the interest earned.

However this action would incur a large cost for the many trips to the bank.

Total Cost

The total cost to the individual is the cost of the trips to the bank plus the interest foregone,

$$\text{Total Cost} = \frac{CT}{W} + \frac{RW}{2}.$$

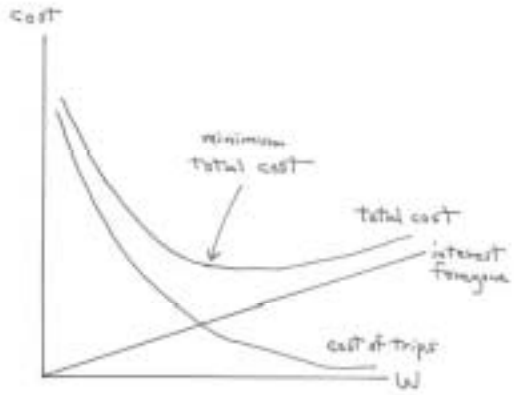
The optimum behavior is to choose W to minimize the total cost.

Figure 2 graphs the two components of the total cost.

The cost of the trips to the bank follows a hyperbola, declining as W increases.

The interest foregone follows a straight line through the origin, as the interest foregone is proportional to W.

Figure 2: Cost



Optimum Withdrawal

The optimum withdrawal occurs when the slope of the total cost is zero.

Setting the derivative of total cost with respect to W to zero gives

$$0 = \frac{\mathrm{d}(\text{Total Cost})}{\mathrm{d}W} = \frac{\mathrm{d}\left(\frac{CT}{W} + \frac{RW}{2}\right)}{\mathrm{d}W} = -\frac{CT}{W^2} + \frac{R}{2}.$$

By the standard calculus rule

$$\frac{\mathrm{d}W^n}{\mathrm{d}W} = nW^{n-1},$$

we have used

$$\frac{\mathrm{d}\left(\frac{1}{W}\right)}{\mathrm{d}W} = -\frac{1}{W^2}$$
$$\frac{\mathrm{d}W}{\mathrm{d}W} = 1.$$

Square-Root Rule

Solving for the optimum withdrawal gives the square-root rule

$$W = \sqrt{\frac{2CT}{R}}.$$

Optimum Demand for Money

Since the average demand for money is $W/2$, the average demand for money is

$$M = \sqrt{\frac{CT}{2R}}.$$

Higher Spending

Higher spending T raises the money demand, but less than in proportion.

Higher Interest Rate

A higher interest rate R reduces the demand for money. To reduce the interest foregone, the individual makes more trips to the bank.

Higher Cost Per Trip

A higher cost per trip C increases the demand for money, as the individual makes a larger withdrawal to reduce the number of trips.

References

[1] William J. Baumol. The transactions demand for cash: An inventory theoretic approach. *Quarterly Journal of Economics*, LXVI(4):545–556, November 1952. HB1Q3.