Labor Productivity and the Real Wage

In the standard Solow model of economic growth, in the long run the economy settles down to steady-state growth, in which labor productivity and the marginal product of labor (the real wage) are constant.

However a stylized fact of growth is that both labor productivity and the real wage grow.

Technical Change

Solow attributes the growth of labor productivity and the real wage to technical advance: holding inputs fixed, output is greater.

Solow adapts his model to allow technical change, by reinterpreting labor. One then carries out the analysis within the same framework.

Aggregate Production Function

A mathematical formulation is to write the aggregate production function not just as a function of capital and labor, but also as a function of time:

$$\widetilde{F}\left(K, L, t\right). \qquad (1)$$

As time passes, product rises, even if the inputs are unchanged.

Cobb-Douglas Production Function

Consider the Cobb-Douglas production function

$$\widetilde{F}\left(K, L, t\right) = \mathrm{e}^{.02t} K^{\frac{1}{3}} L^{\frac{2}{3}}. \qquad (2)$$

Holding capital and labor fixed, the exponential $\mathrm{e}^{.02t}$ means that product grows two *per cent* per year.

In the Cobb-Douglas production function, that the exponents sum to one,

$$\frac{1}{3} + \frac{2}{3} = 1$$

means that returns to scale are constant. In a competitive market economy, the capital share of national income will be $\frac{1}{3}$ and the labor share will be $\frac{2}{3}$.

Labor-Augmenting Technical Change

A special case of the general formula (1) is *labor-augmenting technical change*: one can write the aggregate production function in the form

$$\widetilde{F}\left(K, L, t\right) = F\left[K, A(t)L\right] = F\left(K, L^{*}\right).$$

Effective Labor

Here

$$L^* = A(t)L$$

as *effective labor*. Product is a function of capital K and effective labor L^*.

The term $A(t)$ models the technical change, and one interprets it as an augmentation of the effectiveness of labor. Mathematically, the situation is as if labor becomes more productive with the passage of time, through the term $A(t)$. If A grows by ten *per cent*, then it is as if labor is ten *per cent* more effective.

Cobb-Douglas Example

In the Cobb-Douglas example (2),

$$e^{.02t} K^{\frac{1}{3}} L^{\frac{2}{3}} = K^{\frac{1}{3}} \left(e^{.03t} L\right)^{\frac{2}{3}}. \tag{3}$$

Define effective labor

$$L^* = e^{.03t} L,$$

in which

$$A(t) = e^{.03t}.$$

Production is as if the effectiveness of labor grows by three *per cent* per year.

Aggregate production is

$$F(K, L^*) = K^{\frac{1}{3}} L^{*\frac{2}{3}},$$

a rearrangement of (3).

Growth of Effective Labor

Suppose that labor grows at rate n and that the technical change term A grows at rate g. Then the growth rate n^* of effective labor is sum

$$n^* = n + g.$$

Intensive Form

Instead of working with *per capita* values, work with values per unit of effective labor. For example, define

$$y = \frac{Y}{L^*}$$

and

$$k = \frac{K}{L^*}.$$

The intensive production function is

$$y = f(k).$$

Steady-State Growth

Applying the Solow model, in the long run the economy converges to steady-state growth. The growth rate of effective labor determines the long-run growth rate of the economy. Total values grow at rate n^*, and values per unit of effective labor are constant.

Product Y and capital K grow at rate n^*. Both k and y are constant.

Labor Productivity and the Real Wage

Since labor grows at rate n, productivity (product per worker) grows at rate g.

The marginal product of effective labor is constant. The marginal product of labor (the real wage) therefore grows at rate g.

The stylized facts of long-run economic growth all hold.