Macroeconomics	Technical Change and Long-Run Growth	Macroeconomics	Technical Change and Long-Run Growth
Labor Productivity and the Real Wage		Technical Change	
In the standard Solow model of economic growth, in the long run the economy settles down to steady-state growth, in which labor productivity and the marginal product of labor (the real wage) are constant.		wage to technical adva greater.	rowth of labor productivity and the real ance: holding inputs fixed, output is el to allow technical change, by
However a stylized fact of growth is that both labor productivity and the real wage grow.		-	the then carries out the analysis within
	1		2
Macroeconomics	Technical Change and Long-Run Growth	Macroeconomics	Technical Change and Long-Run Growth
Aggregate Production Function A mathematical formulation is to write the aggregate		Cobb-Douglas Production Function Consider the Cobb-Douglas production function	
production function not just as a function of capital and labor, but also as a function of time:			$(K,L,t) = e^{.02t} K^{\frac{1}{3}} L^{\frac{2}{3}}.$ (2)
$\widetilde{F}(K,L,t)$. (1) As time passes, product rises, even if the inputs are unchanged.		Holding capital and labor fixed, the exponential $e^{.02t}$ means that product grows two <i>per cent</i> per year.	
	3		4
Macroeconomics	Technical Change and Long-Run Growth	Macroeconomics	Technical Change and Long-Run Growth
In the Cobb-Douglas production function, that the exponents sum to one, $\frac{1}{3} + \frac{2}{3} = 1$		Labor-Augmenting Technical Change A special case of the general formula (1) is <i>labor-augmenting</i> <i>technical change</i> : one can write the aggregate production	
means that returns to scale are constant. In a competitive market economy, the capital share of national income will be $\frac{1}{3}$ and the labor share will be $\frac{2}{3}$.		function in the form	$= F[K, A(t)L] = F(K, L^*).$
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$\begin{array}{ccc} \medsimple \text{Merceconomic} \\ \hline \medsimple \medsimple \medsimple \m$				
Here $l^{2} = A(t)l$. as effective labor. Product is a function of capital K and effective labor L'. The deficitive labor L'. The intensive production is as if the effective labor L'. The intensive production is as if the effective labor L'. The intensive production is a deficit labor form L'. The intensive production lis as if the effective labor L'. The intensive production is a deficit labor form L'. The intensive production lis as if the effective labor L'. The intensive production lis as if the effective labor L'. The intensive production lis as if the effective labor L'. The intensive production lis as if the effective labor L'. The intensive production lis as if the effective labor L'. The intensive production lis as if the effective labor L'. The intensive production lis as if the effective labor L'. The intensive production lis as if the effective labor L'. The intensive production lis as if the effective labor L'. The intensive production lis as if the effective labor L'. The intensive production lis as if the effective labor L'. The deficitive labor L'. The defi	Macroeconomics	Technical Change and Long-Run Growth	Macroeconomics Technical Change and Long-Run Growth	
MacroeconomicsTechnical Change and Long-Run GrowthMacroeconomicsTechnical Change and Long-Run GrowthCobb-Douglas Example (2), $e^{02t}K^{\frac{1}{2}}L^{\frac{2}{3}} = K^{\frac{1}{4}}(e^{03t}L)^{\frac{2}{3}}$. (3)Define effective labor $L^* = e^{03t}L$, in which $A(t) = e^{03t}$.Aggregate production is $F(K, L^*) = K^{\frac{1}{2}}L^{*\frac{2}{3}}$, a rearrangement of (3).MacroeconomicsTechnical Change and Long-Run GrowthMacroeconomicsTechnical Change and Long-Run GrowthMacroeconomics <td colspan="2">Here $L^* = A(t)L$ as <i>effective labor</i>. Product is a function of capital <i>K</i> and</td> <td colspan="2">it as an augmentation of the effectiveness of labor. Mathematically, the situation is as if labor becomes more productive with the passage of time, through the term $A(t)$. If A grows by ten <i>per cent</i>, then it is as if labor is ten <i>per cent</i> more</td>	Here $L^* = A(t)L$ as <i>effective labor</i> . Product is a function of capital <i>K</i> and		it as an augmentation of the effectiveness of labor. Mathematically, the situation is as if labor becomes more productive with the passage of time, through the term $A(t)$. If A grows by ten <i>per cent</i> , then it is as if labor is ten <i>per cent</i> more	
Cobb-Douglas ExampleIn the Cobb-Douglas example (2), $e^{\alpha 2r} k^{\frac{1}{2}} L^{\frac{2}{2}} = K^{\frac{1}{2}} (e^{\alpha 3r} L)^{\frac{1}{2}}$. (3)Define effective labor $L^* = e^{\alpha 3r} L$,in which $A(t) = e^{\alpha 3r}$.Production is as if the effectiveness of labor grows by three per cent per year.910MacroeconomicsTechnical Change and Long-Run GrowthHatensive FormInstead of working with per capita values, work with values per unit of effective labor. For example, define $p = n + g$. $n^* = n + g$. $p = f(k)$.		7	8	
$e^{i\Omega t} K^{\frac{1}{2}} f^{\frac{2}{3}} = K^{\frac{1}{3}} (e^{i\Omega t} L)^{\frac{2}{3}}.$ (3) Define effective labor $L^{*} = e^{i\Omega t} L,$ in which $A(t) = e^{i\Omega t}.$ Production is as if the effectiveness of labor grows by three <i>per</i> <i>cent</i> per year. 9 10 Macroeconomics Technical Change and Long-Run Growth Macroeconomics Technical Change Macroeconomics Technical Cha			Macroeconomics Technical Change and Long-Run Growth	
Define effective labor $L^* = e^{0.3t}L,$ in which $A(t) = e^{0.3t}.$ Production is as if the effectiveness of labor grows by three <i>per</i> <i>cent</i> per year. 9 Macroeconomics Technical Change and Long-Run Growth Macroeconomics Technical Change and Long-Run Growth Macroeconomics Technical Change and Long-Run Growth Macroeconomics Technical Change and Long-Run Growth Intensive Form Instead of working with <i>per capita</i> values, work with values <i>per unit of effective labor.</i> For example, define $y = \frac{Y}{L^*}$ and $k = \frac{K}{L^*}.$ The intensive production function is y = f(k).	In the Cobb-Douglas e	xample (2),		
in which $A(t) = e^{-\Omega t}$.a rearrangement of (3).Production is as if the effectiveness of labor grows by three percent per year.910MacroeconomicsTechnical Change and Long-Run GrowthMacroeconomicsTechnical Change and Long-Run GrowthMacroeconomicsTechnical Change and Long-Run GrowthMacroeconomicsTechnical Change and Long-Run GrowthMacroeconomicsTechnical Change and Long-Run GrowthIntensive FormSuppose that labor grows at rate n and that the technical change term A grows at rate g. Then the growth rate n* of effective labor is sumInstead of working with per capita values, work with values per unit of effective labor. For example, define $y = \frac{Y}{L^*}$ and $k = \frac{K}{L^*}$.The intensive production function is $y = f(k)$.	Define effective labor			
Production is as if the effectiveness of labor grows by three per cent per year.10MacroeconomicsTechnical Change and Long-Run GrowthMacroeconomicsTechnical Change and Long-Run GrowthMacroeconomicsTechnical Change and Long-Run GrowthMacroeconomicsTechnical Change and Long-Run Growth Growth of Effective Labor Suppose that labor grows at rate n and that the technical change term A grows at rate g. Then the growth rate n* of effective labor is sumInstead of working with per capita values, work with values per unit of effective labor. For example, define $y = \frac{Y}{L^*}$ and $k = \frac{K}{L^*}$. The intensive production function is $y = f(k)$.	in which		a rearrangement of (3).	
MacroeconomicsTechnical Change and Long-Run GrowthMacroeconomicsTechnical Change and Long-Run Growth Growth of Effective Labor MacroeconomicsTechnical Change and Long-Run GrowthSuppose that labor grows at rate n and that the technical change term A grows at rate g . Then the growth rate n^* of effective labor is sumMacroeconomicsTechnical Change and Long-Run Growth $n^* = n + g$.MacroeconomicsTechnical Change and Long-Run Growth $n^* = n + g$.MacroeconomicsTechnical Change and Long-Run Growth $prove = \frac{Y}{L^*}$ Instead of working with $per capita$ values, work with values per unit of effective labor. For example, define $y = \frac{Y}{L^*}$ and $n^* = n + g$. $k = \frac{K}{L^*}$.The intensive production function is $y = f(k)$.	Production is as if the effectiveness of labor grows by three <i>per cent</i> per year.			
Growth of Effective Labor Suppose that labor grows at rate <i>n</i> and that the technical change term <i>A</i> grows at rate <i>g</i> . Then the growth rate n^* of effective labor. For example, define $y = \frac{Y}{L^*}$ and $k = \frac{K}{L^*}$. The intensive production function is $y = f(k)$.		9	10	
Growth of Effective Labor Suppose that labor grows at rate <i>n</i> and that the technical change term <i>A</i> grows at rate <i>g</i> . Then the growth rate <i>n</i> [*] of effective labor. For example, define $y = \frac{Y}{L^*}$ and $k = \frac{K}{L^*}$. The intensive production function is $y = f(k)$.	Macroeconomics	Technical Change and Long-Run Growth		
Growth of Effective Laborper unit of effective labor. For example, defineSuppose that labor grows at rate n and that the technical change term A grows at rate g. Then the growth rate n^* of effective labor is sum $y = \frac{Y}{L^*}$ $n^* = n + g$.and $k = \frac{K}{L^*}$.The intensive production function is $y = f(k)$.			Intensive Form	
term A grows at rate g. Then the growth rate n^* of effective labor is sum $n^* = n + g$. and $k = \frac{K}{L^*}$. The intensive production function is y = f(k).	Growth of Effective Labor			
y = f(k).	Suppose that labor grows at rate n and that the technical change term A grows at rate g . Then the growth rate n^* of effective labor is sum		and $k = \frac{K}{L^*}.$	
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	11		12	

Macroeconomics	Technical Change and Long-Run Growth	Macroeconomics	Technical Change and Long-Run Growth
Applying the Solow m converges to steady-sta labor determines the lo Total values grow at ra labor are constant.	ady-State Growth odel, in the long run the economy ate growth. The growth rate of effective ong-run growth rate of the economy. te n^* , and values per unit of effective K grow at rate n^* . Both k and y are	Since labor grows at a grows at rate g. The marginal product marginal product of la rate g.	luctivity and the Real Wage rate <i>n</i> , productivity (product per worker) t of effective labor is constant. The abor (the real wage) therefore grows at long-run economic growth all hold.
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