Eliminating Substitution Bias

One eliminate substitution bias by continuously updating the market basket of goods purchased.

Two-Good Model

Consider a two-good model. For good i, the price is p_i, and the quantity demanded is q_i. The total cost is

$$c = p_1 q_1 + p_2 q_2.$$

Minimum Cost

Consider the minimum cost c of attaining an indifference curve, a function of the prices of the two goods. Consider particular prices p_i and cost-minimizing quantities q_i. By definition, the minimum cost for these prices is

$$c = p_1 q_1 + p_2 q_2.$$

By the concept of the consumer price index, the rate of inflation is the rate of change in the minimum cost of attaining the indifference curve.

Change in Cost

If the prices change by small amounts, then the change Δc in the minimum cost is

$$\Delta c \approx q_1 \Delta p_1 + q_2 \Delta p_2. \tag{1}$$

We prove this result below.

Fractional Change in Cost

The fractional change in the cost is therefore

$$\frac{\Delta c}{c} = \frac{q_1 \Delta p_1 + q_2 \Delta p_2}{c}$$
$$= \left(\frac{p_1 q_1}{c}\right) \frac{\Delta p_1}{p_1} + \left(\frac{p_2 q_2}{c}\right) \frac{\Delta p_2}{p_2}$$

On the right-hand side, each term shows the contribution to inflation of the price change for each good.

Budget Shares

The expression $p_i q_i / c$ is the budget share for the good, the fraction of total spending on the good. The contribution to inflation from a good is its budget share multiplied by its fractional change in price.

By definition, the sum of the budget shares for all goods is one. Thus the rate of inflation is the weighted average of the rates of inflation on the different goods, using the budget shares as weights.

In the computation of inflation, the inflation rate for a good with a low budget share receives low weight.

Cost of a Market Basket

We derive the change in minimum cost (1) by a graphical argument.

Consider a market basket A of goods on the indifference curve, with the quantities q_i (figure 1). Taking the price p_2 of good 2 as fixed and letting the price p_1 of good 1 rise, the cost of A rises, with slope q_1 and intercept $p_2 q_2$ (figure 2).

Figure 1: Indifference Curve

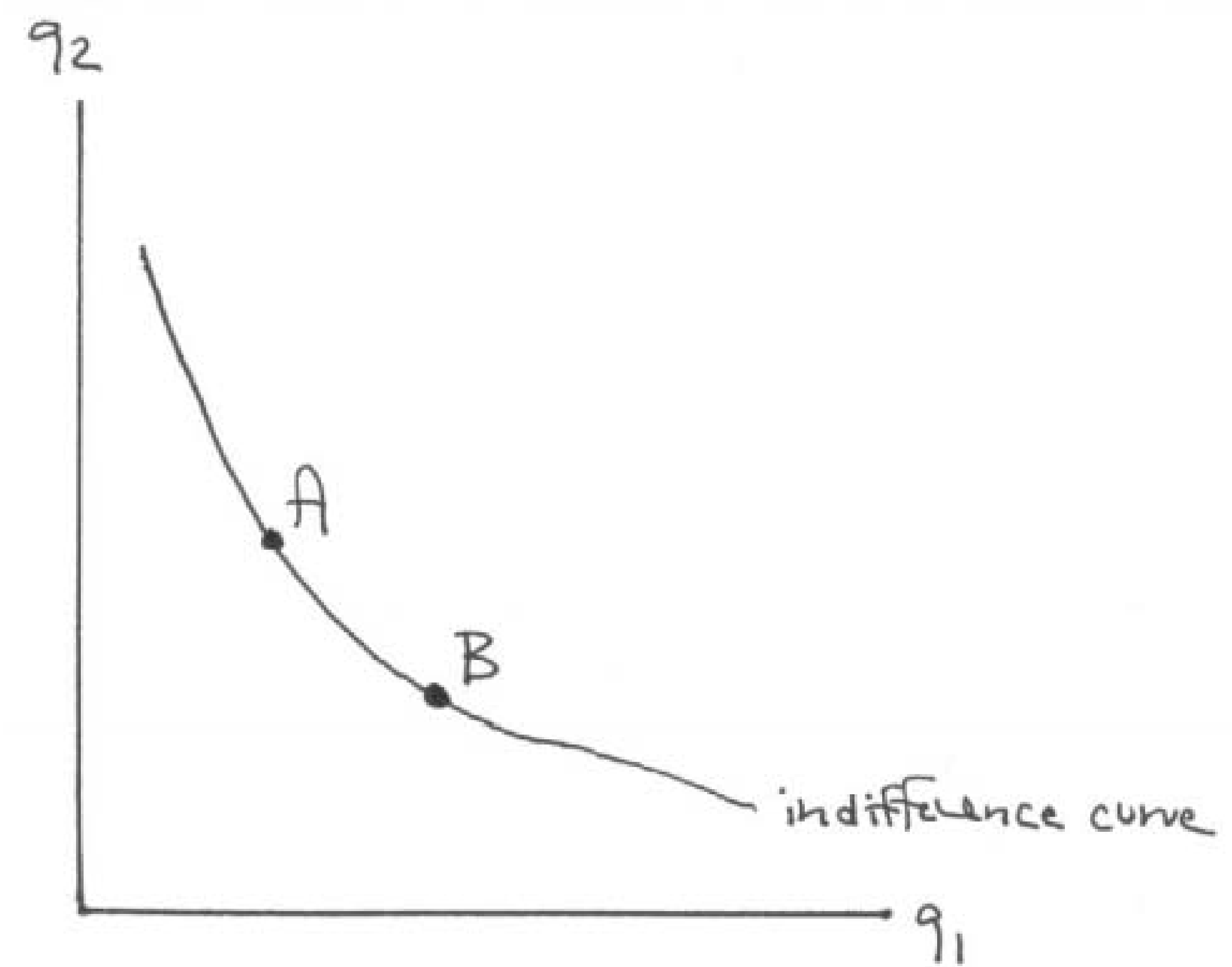
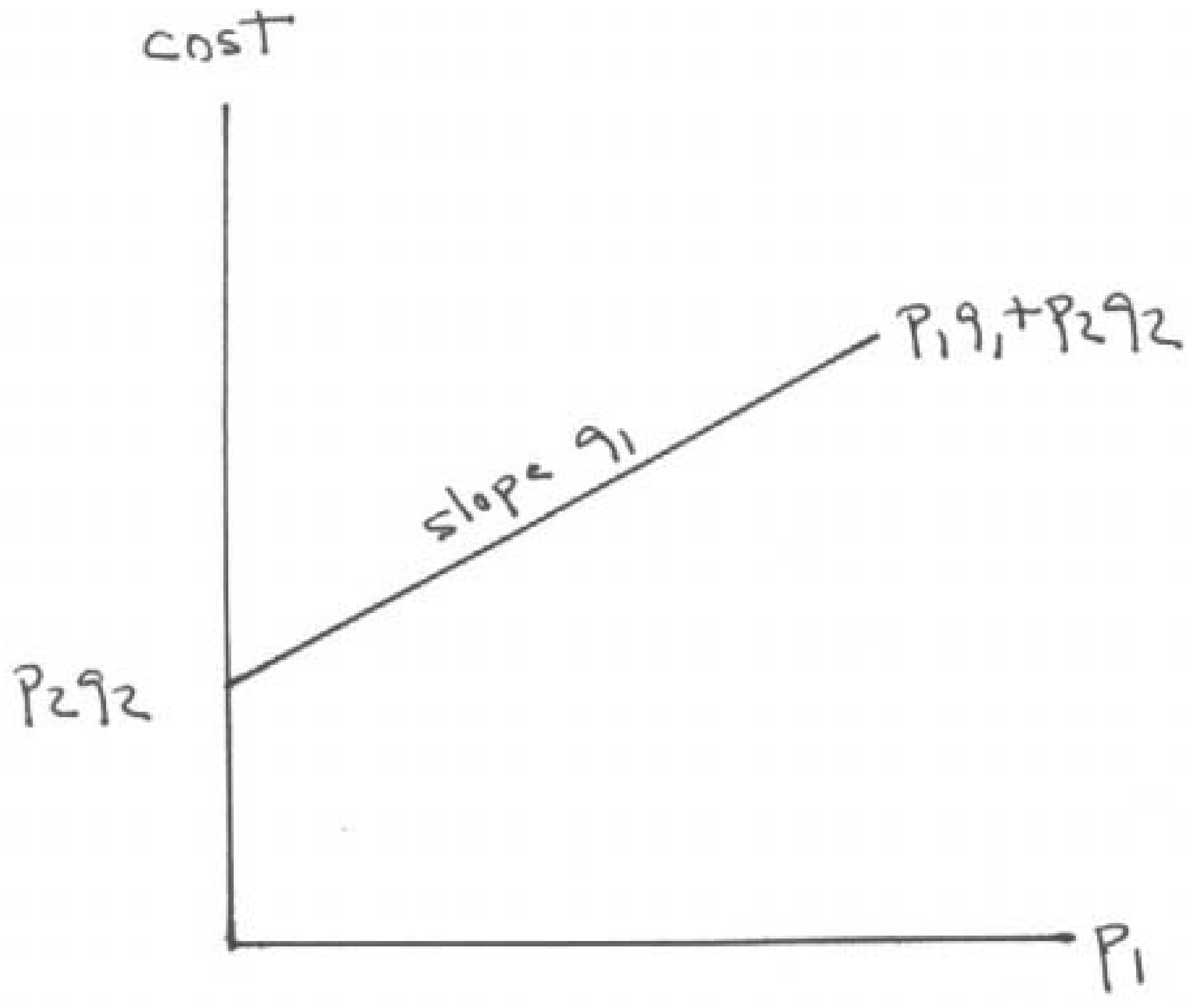


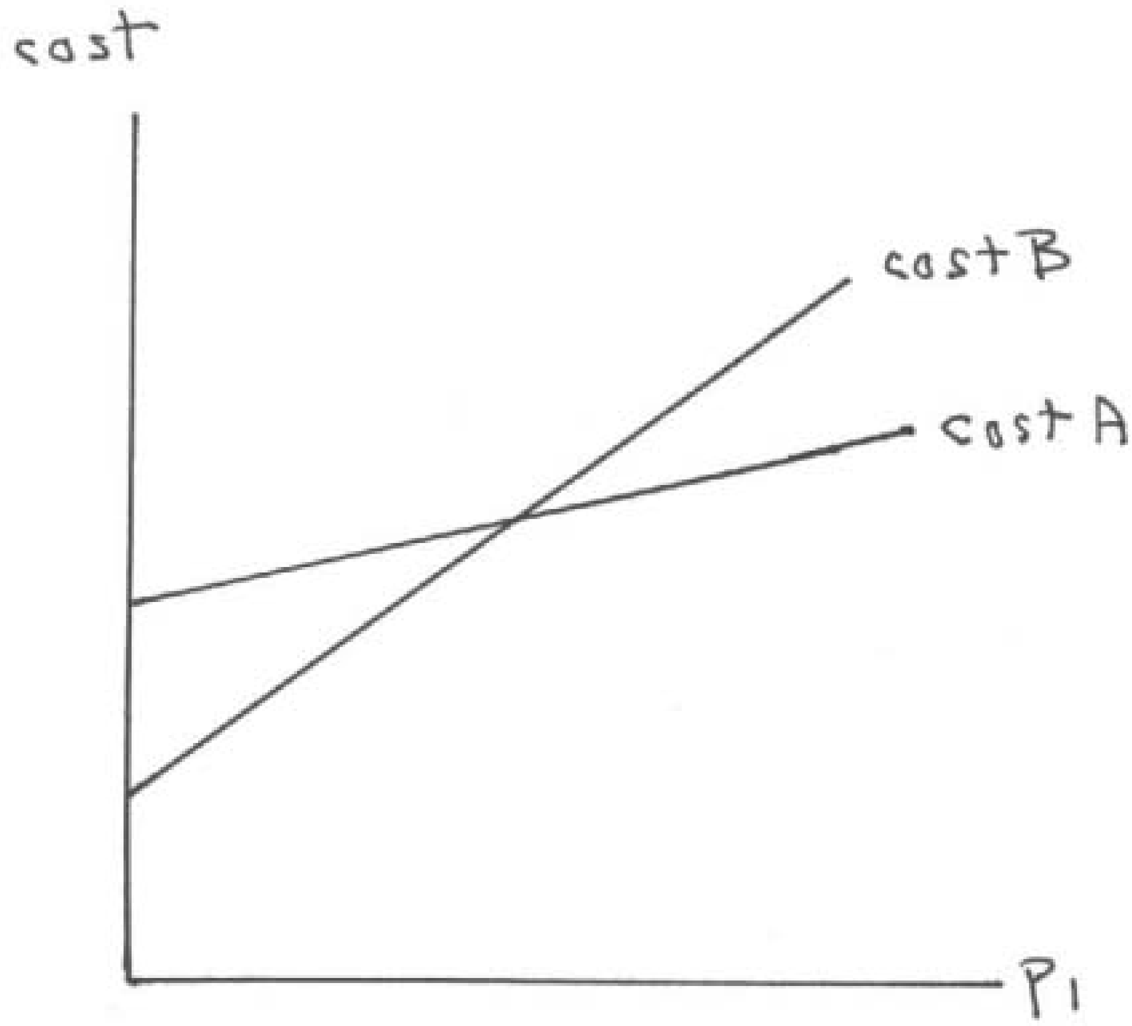
Figure 2: Cost of a Market Basket



Alternatively, consider the market basket B of goods, also on the indifference curve. Taking the price p_2 of good 2 as fixed and letting the price p_1 of good 1 rise, the cost of B is again a straight line (figure 3).

Relative to market basket A, for B the quantity of good 1 is larger and the quantity of good 2 is smaller. For small p_1 the cost of B is less than the cost of A; for large p_1 the cost of B is higher.

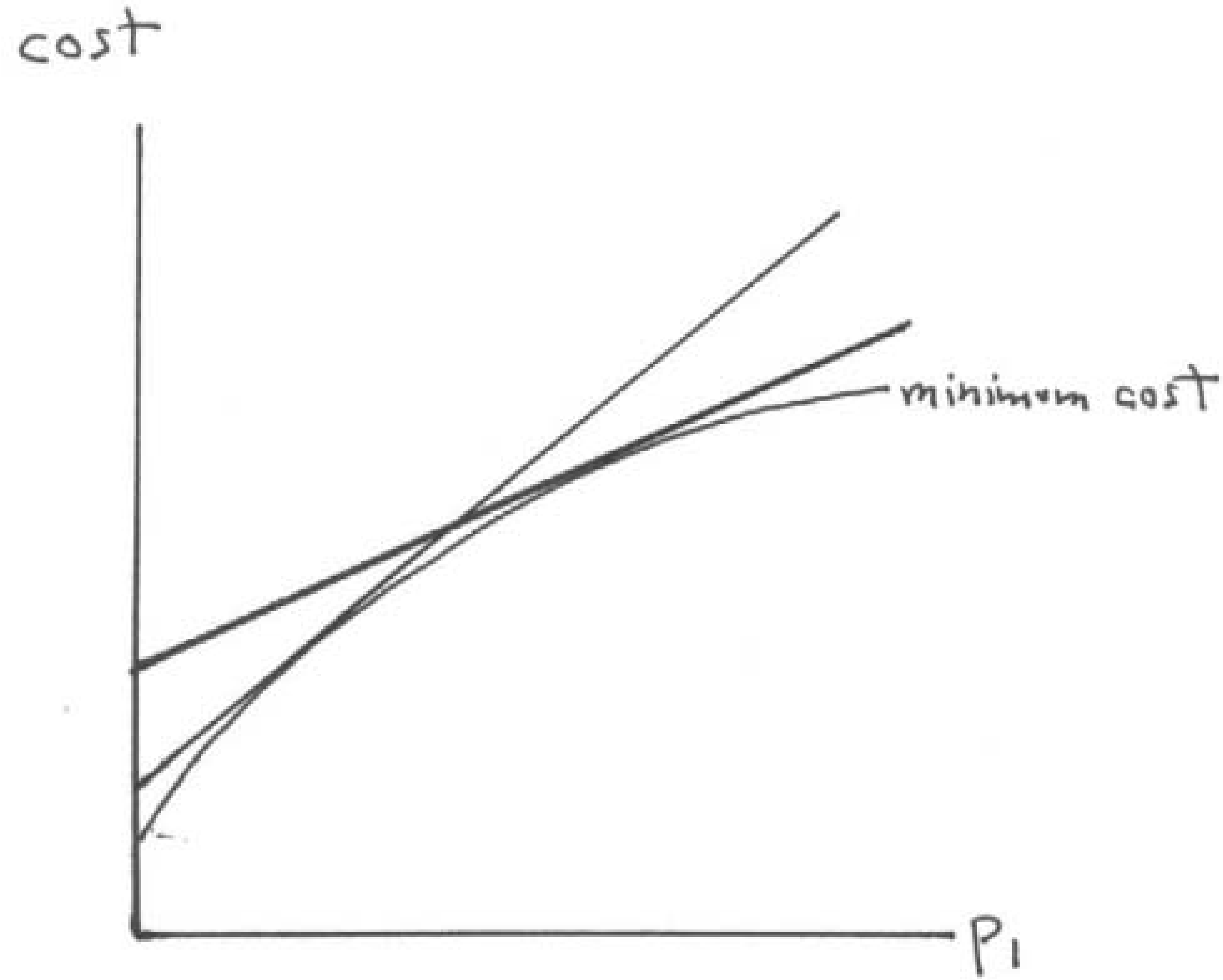
Figure 3: Alternative Market Baskets



Minimum Cost as Lower Envelope

The minimum cost as a function of p_1 is the lower envelope of the straight lines showing the cost of each market basket (figure 4).

Figure 4: Minimum Cost

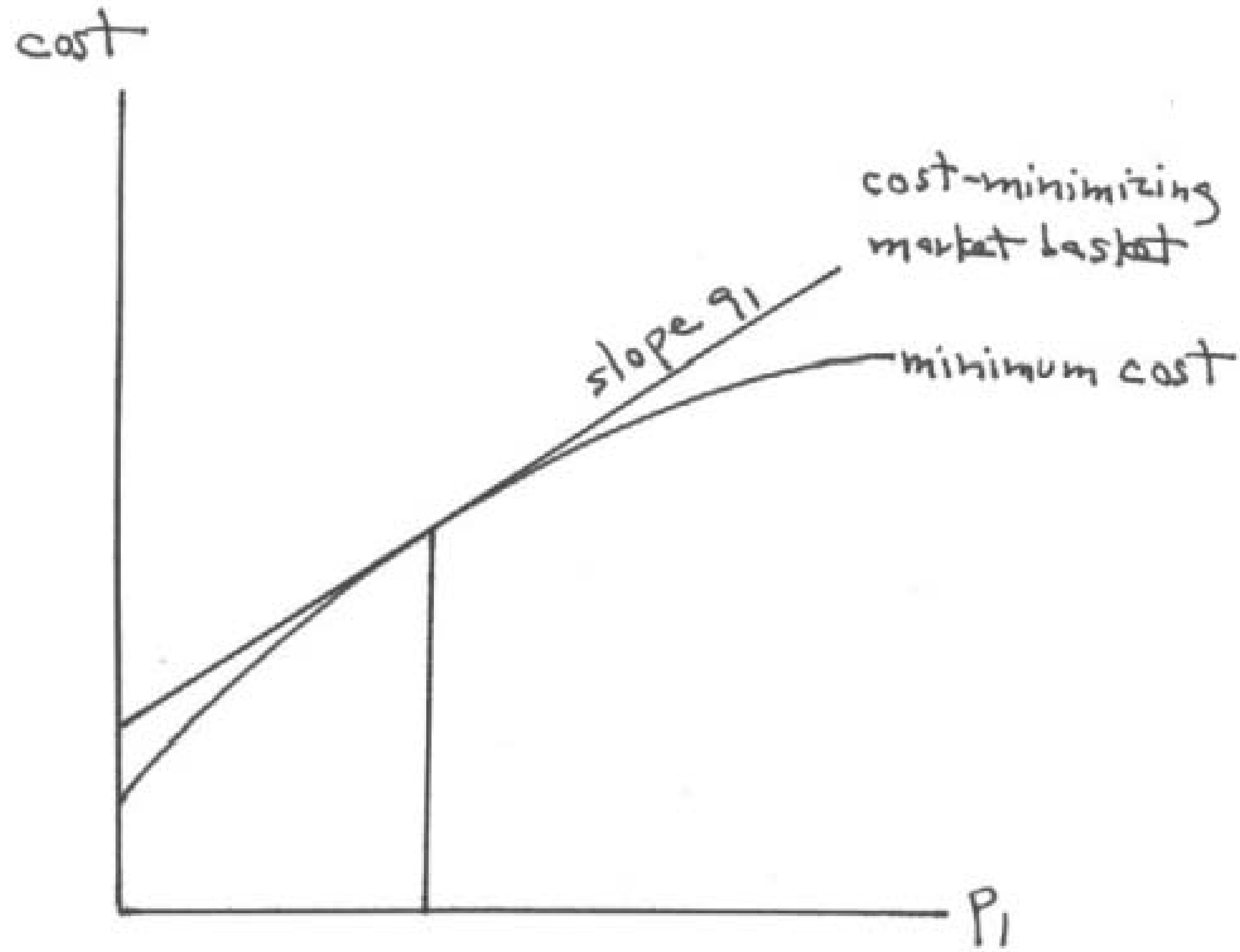


Envelope Theorem

For any particular p_1, some market basket of goods on the indifference curve has the minimum cost (figure 5). The straight-line cost for this market basket must be *tangent* to the minimum cost curve: the straight line and the curve touch at this point, and for the straight line to cross the minimum cost curve would contradict the definition of the latter.

Tangency means that both have the same slope. The slope of the straight line is q_1, the quantity of good 1 in the cost-minimizing market basket, so the slope of the minimum cost curve is also q_1. This tangency result is the *envelope theorem*.

Figure 5: Envelope Theorem



Thus, for small changes in p_1, the change in the minimum cost is

$$\Delta c \approx q_1 \Delta p_1. \tag{2}$$

For small changes, the straight line shows the change in cost.

The relationship (1) follows, as the same argument can be made for a change in the price of the other good.

Large Changes in Price

For changes in the price p_1 that are not small, the straight line does *not* show the change in cost. The minimum cost rises *less* that the cost of the fixed market basket at the tangent point (figure 5).

Changing the market basket lowers the cost; this relationship is just the substitution bias.

Eliminating Substitution Bias

Pricing a *fixed* market basket is the source of substitution bias. Formula (1) implies that one eliminate substitution bias by continuously updating the market basket of goods purchased.