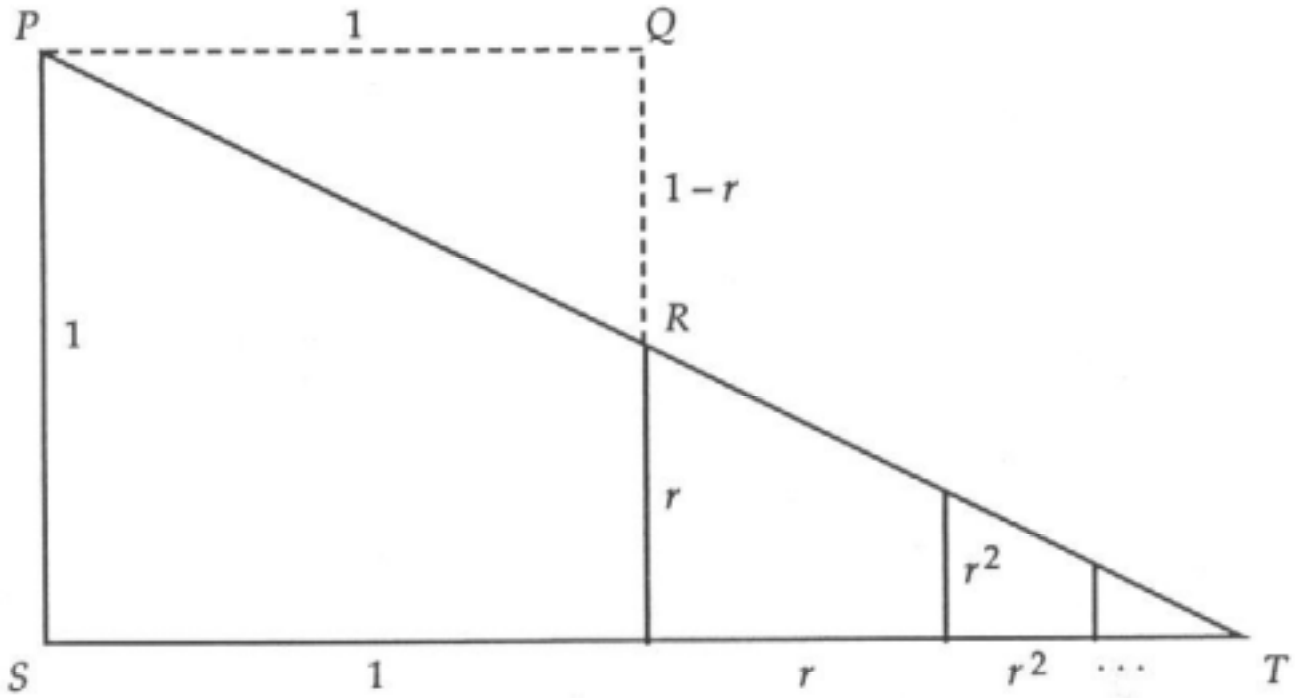
Infinite Geometric Sum

Some basic mathematical formulas can be proved by a geometric "proof without words" [1]. Figures 1 and 2 each prove the infinite geometric sum formula

$$\frac{1}{1-r} = 1 + r + r^2 + \cdots .$$

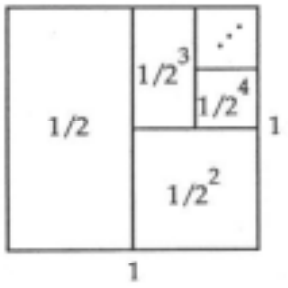
Figure 1: Benjamin G. Klein and Irl C. Bivens



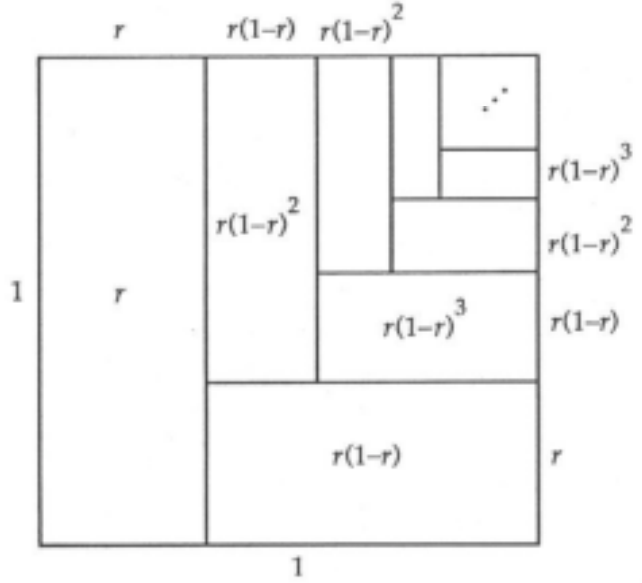
$$\Delta PQR \approx \Delta TSP$$

$$\therefore 1 + r + r^2 + \ldots = \frac{1}{1 - r}.$$

Figure 2: Warren Page



$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 1$$



$$r + r(1-r) + r(1-r)^2 + \cdots = 1$$

References

[1] Roger B. Nelsen. *Proofs Without Words: Exercises in Visual Thinking*. The Mathematical Association of America, Washington, DC, 1993. QA90N385 1993.