A Detailed Teaching Philosophy
and Historical Portfolio

of

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Introduction and Outline of Chapters

The following should help to provide a brief overview of my personal beliefs, common practices, and professional history, with regards to teaching at the post-secondary level. The bulk of the portfolio is found in section one, which details my general teaching philosophy, and section two, covering general course development and in-class practices. Section three mentions a few opportunities that I have taken to both improve upon my own instruction and assist in the professional development of my colleagues. The last section demonstrates my teaching successes, and provides some statistics regarding my overall effectiveness as an instructor. The appendices present a sample syllabus, as well as two exams from courses taught in the 2009-2010 academic year.

Contents

1 Teaching Philosophy

Since 2005, I have been an instructor at the University at Albany for ten courses, spanning five subjects, with enrollments as small as ten and as large as sixty students. My responsibilities as an instructor have been identical to those of the department's full-time faculty, and include the standard tasks of writing a syllabus, maintaining a course website, holding weekly office hours, assigning homework, composing and administering exams, and evaluating student performance. In addition to these basic duties, I also provide myself with detailed lesson plans, as it is my personal belief that there is a direct correlation between an instructor's overall effectiveness in the classroom and her or his preparedness for each lecture. Lastly, I appreciate the use of relevant technological aids in the classroom, and make every effort to incorporate them into course lectures and assignments.

In regards to my overall teaching philosophy, it is the hope that a balance is struck between the level of care that I show for my students and the demands that I impose upon them. Coupled with course material, the central themes that I try to convey are humility, confidence, willingness, and responsibility. Additionally, I encourage my students to prioritize understanding ahead of simply memorizing and employing an algorithm, since figuring out why a given method works is always more enlightening than using the method to obtain a result. These personal beliefs and core values have developed as a result of my own education and personal success, and in many cases, they extend beyond the realm of the classroom.

Learning mathematics requires equal amounts of humility and confidence. In order to achieve success, students must acknowledge early on that although the material is rarely easy to learn, it is never beyond their capabilities. A minimum level of enthusiasm is also essential, as well as a readiness of the instructor to provide assistance and encouragement. Consequently, I often urge my students to speak up in class and attend my weekly office hours. I stress that office hours are usually the best environment to receive assistance, and I make every attempt to be both receptive and accommodating to those individuals who may require extra help, but cannot attend my usual times. My willingness to help students outside of the classroom is, without a doubt, one of my strongest assets, and I believe it has helped to breed a mutual respect between student and instructor over the years.

In addition to the aforementioned values, a certain level of responsibility must be expected from both the students, and their instructor. I take a great deal of pride in my role as an instructor, and work hard to maintain the level of professionalism that the position demands. Similarly, it is critical that each student understands exactly what is expected of her or him, both inside of the classroom and while enrolled at a particular institution. As a result, I try to instill in each individual my view of an undergraduate education as a maturation process, where the student is solely responsible for understanding school policies and fulfilling all course requirements.

To teach at any level, particularly a post-secondary one, is a great privilege that allows for experimentation of different methods, yet demands constant self-evaluation. Each new term presents the opportunity to both establish and sustain a positive academic relationship with each individual, and requires an equal amount of commitment and hard work from both parties. My history as an instructor at the University at Albany is a testament to the success that I have had in building these relationships over the years.

2 Course Materials and Curriculum

2.1 The Syllabus

Naturally, a course syllabus should be presented to students at some point during, or possibly before the first lecture or meeting time, and contain every bit of information relevant to the course. Most importantly, a syllabus should give students a clear description of all student requirements, including tentative dates and deadlines for exams and assignments. My syllabus format has evolved from former teaching assistants and colleagues over the years, and has since become the model for a number of TAs and new instructors at UAlbany.

Besides the requirements outlined in the University syllabus guidelines, I always take a moment to consider an overall course objective, in which I briefly state my aspirations for each student over the next few months. Additionally, my syllabus usually includes tutoring options that are available to the students, both through the department and outside organizations, and I attach a calendar to each syllabus, listing all exams and school recess dates. And although the optimist would like to ignore it, the realist in me makes certain to inform the students of the critical withdrawal deadline, so anyone who is in jeopardy of failing is not surprised when the date passes.

The syllabus from a Linear Algebra course taught in the Fall of 2009 is included as an appendix. This particular syllabus, along with those from every course I have ever taught, is available at my website. The changes and inclusions from each semester demonstrate how the general format has evolved, and also help to exhibit my experimentation with homework assignments, exams, student presentations, WebAssign and other course technology, as well as overall grading percentage breakdowns.

2.2 Homework, Presentations, and Course Accompaniments

My stance on homework has changed wildly over the years, and the percentage that homework contributes to a course's final grade has been as low as zero and as high as sixty. While I still have not perfected a science for determining the appropriate percentage that homework should be worth, I will continue to consult with colleagues and experiment with both the amount and how often it is assigned, as well as my methods for grading. My decisions regarding homework submission and grading follow closely with the beliefs outlined in my personal teaching philosophy. Simply stated, homework helps, and a student's realization of this fact is part of what I refer to as the "undergraduate maturation process".

I also have had success with encorporating web-based tutorials such as WebAssign into a student's overall homework grade, and reducing the high levels of frustration that many students experience, when having to submit homework assignments through a computer. And although I constantly seek to dissolve student dependence on calculators, I still acknowledge them as a valuable learning aid and encourage their use, especially as a means of checking one's work.

Furthermore, I find individual student projects and presentations to also be very beneficial, as a way to demonstrate the real world applications of a specific topic, and break the monotony of simply taking notes, reading the textbook, and doing homework. Additionally, they allow for a temporary, yet appropriate increase in student responsibility, and can be particularly useful to those students who may be entering a profession that requires them to talk in front of an audience.

For two semesters, I required a student presentation, as part of a basic course in Linear Algebra. The presentations were chosen from a list of textbook problems, which were selected either due to an increased level of difficulty or the requirement of math computational software (Maple, MATLAB, Sage, Mathematica, etc.). Students were encouraged to approach me for help, and would present their solutions in my office prior to presenting in class, in order to alleviate any nervous feelings. The inclusion of this requirement was a success for a number of reasons, most notably because it proved a nice introduction to the power of technological aids in mathematics and statistics, as most students had little or no prior exposure to the software.

Lastly, as an added effort to maintain a high level of professionalism, I also provide my students with a course website, where they can find my school contact information and access a copy of the syllabus. During the semester, the site also includes a regularly updated list of suggested homework and relevant presentations, as well as a spreadsheet that privately shows all individual grades. The grades spreadsheet also allows students to ”pro-rate" their end-of-semester final grade at any point during the semester, which many students have found useful in helping to keep them on pace for achieving their desired final grade.

2.3 Quizzes and Exams

I have found that frequent 10-15 minute quizzes are extremely effective in preparing students for exams and keeping them up-to-date with course content, while not stealing away too much time from lectures. Furthermore, I am a proponent of giving a minimum of ten quizzes per semester, and dropping the lowest one or two quizzes from a student's overall average. I believe that the ability to ”scrap" a score helps to lower stress levels during each quiz, and allows for flexibility, if the student simply cannot devote the amount of time that she or he would like to my course during a particular week. Moreover, my decision to not give ”pop" quizzes reflects my attempt to build a mutual level of student-instructor respect. I also strongly advocate the use of quizzes for testing student recollection of definitions and theorems, whose understanding is vital to so many higher-level mathematics courses.

A minimum of three examinations should make up a large portion of a student's overall grade, but no single exam should ”make or break" their semester. I believe that many instructors often place too much emphasis on a mid-term or final exam, creating unusually high levels of stress on an already over-stressed student. While homework alone does not provide a completely accurate student assessment, the pressure that a timed in-class exam creates can often adversely affect individual performance. One way that I seek to quell this is by administering three to four exams and avoiding the terms ”mid-term" and ”final" altogether.

Two sample exams are provided as appendices at the end of this portfolio, to help demonstrate the careful consideration that I put into the creation of each exam. To paraphrase Donald Wilken, a personal mentor and professor at UAlbany, an effective exam should show how much a student knows, as well as some things they *may not* know, or more specifically, what concepts they might not fully understand. For every course, regardless of the number of times I have taught the subject, I work hard to create a unique exam that does not intimidate test-takers and effectively tests *both* understanding of theory and application of techniques. Consequently, I inform my students that being able to correctly answer every textbook problem in a particular section does not necessarily reflect complete understanding of the theory from the section.

Perhaps more important than the composition of an exam, is the grading and analysis of student performance. The evaluation of exams should take no longer than one week and provide quality student feedback and explanations regarding any deductions. I make every effort to meet both of these requirements, usually giving a thorough in-class analysis of the exam no later than two meetings after the exam is administered. This analysis includes providing solutions for all problems, either as a handout or in class, explaining the intent associated to each question, and presenting a complete breakdown of grades, detailing average scores for individual questions. The time spent going over the exam not only helps to calm angry or frustrated students, it also creates a sense of closure, enabling the class to focus on future topics. Most importantly, the grading breakdown acts as a personal quality control, helping me to identify which concepts might require additional treatment at the end of the semester or what specific exam questions may have been poorly constructed.

2.4 Lectures and Office Hours

Both my personal teaching philosophy and my student evaluations provide sufficient evidence, supporting my policy regarding office hours. Typically, students requiring extra help are expected to attend those office hours specified in the course syllabus, but I am always receptive to those individuals who have conflicting schedules, and I rarely keep my door closed to my students. Furthermore, no student should be reluctant or afraid of asking for help from their instructor, and I make every effort to come off as both a supportive and encouraging teacher for *all* of my students.

Contrary to some colleagues, I neither support nor condone ”winging" a lecture, and regardless of prior experience, I will continue to prepare detailed lecture notes for each course that I teach. Creating notes helps me to determine the appropriate organization of theory with the inclusion of sufficient examples, and personally calms nerves and builds my confidence prior to teaching. As an added benefit to my students, most of the examples that I present during lectures are problems that have not been solved in the textbook, and are specifically chosen for this reason.

Typically, I allocate ten to fifteen percent of the total class time for student questions. Usually this takes place prior to the administering of a quiz or the start of a lecture. In the event that I am concerned about having enough time to cover a specific topic, I will sometimes ask that students hold their questions until right before the end of class. And even when there are no questions, I encourage students to ask me anything regarding the previous lecture's topic, and will often pose my own questions, if they still have none. I prefer this approach to simply presenting a lecture, as it helps reduce boredom levels, unite the class, and increase student interest and participation. The opportunity to ask a question in class, rather than office hours, is often greatly appreciated by those students who do not require a great amount of personal assistance.

I would like to believe that the most important aspect of my lectures is the approach that I take presenting the theory behind the examples that I provide for my students. "Understanding" is a term that is used frequently in this portfolio and during lectures, and I often preach to my students the mathematical power behind this word. Many of my students will attest to my being quoted as saying I simply do not care what the answer to a problem is, but rather the steps that one takes and the the theory that is used to get to the answer. I relate to my students that this aspect of mathematical learning was never truly embraced by me as an undergraduate major, and I consequently missed out on understanding the "big picture", having been relegated to the role of a "human calculator". Both my students' end-of-semester comments and renewed success in the field, even after they have completed my class, suggest that I have been effective in taking this approach.

3 Departmental Participation and Professional Development

Most of my professional development experiences have been provided in connection with my time spent as a graduate assistant under the federal research grant titled *Teaching Mathematics for Understanding.* This grant presented me with a unique opportunity to suspend my basic duties as a teaching assistant, and work alongside local area middle and secondary school teachers. As an assistant, I both administered and actively participated in mathematics workshops and worked with the middle school teachers to help develop and improve upon their course curriculum. As an added benefit, I became enrolled as a member of the *Association of Mathematics Teachers of New York State*, and attended the statewide conference on teaching and learning in the Winter of 2007. Without a doubt, my involvement with the grant opened my eyes to the methods of effective instruction, and sparked my interests in taking advantage of future professional development opportunities.

After resuming normal teaching assistant duties with the department of Mathematics and Statistics in the Fall of 2008, I began to take an interest in coming up with ways that I could help my fellow teaching assistants improve upon their own teaching practices. Consequently, as the newly appointed coordinator of the department's Graduate Student Seminar, I often utilized the seminar in this regard, giving talks on course website creation and the use of LaTeXfor syllabus and exam creation. I also held discussions regarding personal teaching strategies for the various courses assigned to the department's TAs.

These discussions eventually resulted in my request to create the departmental position of Tutoring Room Supervisor, which I held during the 2009-2010 academic year. As Supervisor, I continued to hold workshops and act as a mentor for both undergraduate student tutors and young teaching assistants assigned to the room, often giving advice on teaching and student assessment. Many of these TAs had little or no teaching experience prior to this, and I would like to think that my role aided in founding their careers as educators.

In the Spring of 2010, I also contributed the department beyond my standard teaching assistant responsibilities by coordinating and composing a department-wide final exam and facilitating an all-day review session for students from at least seven sections of Calculus II. This small, yet valuable voluntary contribution not only speaks to my responsibility and leadership qualities, but also the vested interest that I place in the department, its proceedings, and the success of its students.

4 Teaching Honors and Student Comments

The Excellence in Teaching award is the only formal honor bestowed upon the teaching assistants in the Department of Mathematics and Statistics at the University at Albany. The award is typically given to two TAs at the end of the academic year. Recipients are agreed upon by the faculty, and decisions are usually made based on the instructor's end-of-semester student evaluations. The award carries with it a small monetary sum, but the honor in being chosen is usually the real prize, since it often marks the first award of its kind for young instructors, and acts as a confirmation of her or his effectiveness in the profession. For these very reasons, earning the award has always been one of my professional goals, and in the Spring of 2010 I achieved this goal.

Listed below are a few of the positive comments that I have received in my course evaluations over the years. In addition to the personal morale booster, these evaluations have been extremely useful in helping me improve upon my teaching practices.

> The instructor was always well-prepared for class and communicated course content the best out of any of my T.A. instructors. I was definitely challenged intellectually in this class and was treated pretty fairly in grading on exams, homework, and quizzes. The instructor made himself available for my needs out of class every time that I needed him to. This was probably his best and most effective attribute. He did a real good job.

> I've had a poor background in pre-calc, but he was able to make me understand. Still, he was always available for extra help. The course was taught at a fair (but not easy) level. Easy to understand. Quizzes and tests were set up so that grading was fair (quizzes could help grade, but doing poorly on one test or quiz would not ruin grade). Good amount of material covered. Thanks to the professor I no longer hate math :)

> Your enthusiasm was contagious; always excited to teach. Pretty effective in keeping things interesting. Calculus isn't always the most exciting subject! Very fair in grading. Always well prepared with your notes. WebAssign was frustrating, but you addressed it well.

> I think that even though I don't really enjoy math, I learned quite a bit from the instructor. This is my second time taking math. I failed it two semesters ago because the professor was incoherent and didn't teach the subject matter that well. I may not do well in this class because of the fact that I did not attend regularly, but when I was here, he taught it very well and was very attentive.

Regarding professionalism

- Always willing to help.
- Made sure to be available outside of class, not only with office hours, but also additional sessions if it was necessary.
- Returned quizzes/tests very quickly.
- Quick to respond to emails.
- Gives a lot of work.
- Gave clear notes, always spoke effectively and stayed on track.
- Whenever someone had a question, he always did his best to answer.
- Makes an effort to present things in ways we can understand.
- Sometimes stumbled over material and made it confusing, but would re-do the lecture the next class. Good office hours. During lecture I don't think that all the material he said had to be written on the blackboard, but I guess that makes his lecture thorough.
- He is the perfect instructor for lectures like mathematics, that require a lot of understanding.

Regarding character

- Great energy with class.
- Always happy to teach.
- Very dedicated.
- Always able to keep the class interesting.
- Sincerely cares about every one of his students.
- Really helped me believe that anyone could learn math.
- Actually made calculus kind of fun.

4.1 Student Evaluation Summary

Course Evaluation

Rating	Fall 08* Calc II	Summer 09 Calc I	Summer 09 Lin Alg	Fall 09 Lin Alg	Spring 10 Calc II	Total
5-Highest	8	5	4	9	8	34
4	9	5	5	4	9	32
3	2	1	5	0	3	11
2	0	0	0	2	0	2
1	0	0	0	1	0	1
0-Lowest	0	0	0	0	0	0
Total	19	11	14	16	20	80
% Responding	66	79	82	84	83	78
Mean	4.32	4.36	3.93	4.13	4.25	4.20

*No departmental data collected prior to Fall 2008

Instructor Effectiveness

Rating	Fall 05* Alg & Calc	Spring 06 Alg & Calc	Fall 07 Survey of Calc	Spring 08 Calc I	Fall 08 Calc II	Summer 09 Calc I	Summer 09 Lin Alg	Fall 09 Lin Alg	Spring 10 Calc II	Total	Total Sp06-Sp10
5-Highest	0	8	8	6	8	8	6	10	13	67	67
4	8	10	7	4	9	2	7	3	6	56	48
3	7	9	4	4	2	1	1	0	1	29	22
2	2	1	2	0	0	0	0	3	0	8	6
1	2	0	1	0	0	0	0	0	0	3	1
0-Lowest	1	0	1	0	0	0	0	0	0	2	1
Total	20	28	23	14	19	11	14	16	20	165	145
% Responding	83	88	43	58	66	79	82	84	83	74	68
Mean	2.95	3.89	3.70	4.14	4.32	4.64	4.36	4.25	4.60	4.03	4.18

*First teaching experience

A Sample Syllabus

AMAT 220: Linear Algebra
Class #19412 Fall 2009

Instructor: Benjamin Atchison, T.A. Office: ES 151A
Email: ba577642 (@albany.edu) Phone: (518) 442-4644
Meeting Time/Place: T/TH 8:45-10:05 in ES 143
Office Hours: T/TH 10:15-11:45 in ES 151A and by appointment
Class Websites: www.albany.edu/∼ba577642
Text: Lay, David C. Linear Algebra and Its Applications. 3^{rd} Edition.
Pearson/Addison-Wesley. 2006. ISBN: 0-321-28713-4.

Bulletin Description: Linear equations, matrices, determinants, finite dimensional vector spaces, linear transformations Euclidean spaces. Prerequisite(s): A Mat 113. Taken from the MyUAlbany Catalog Detail Description.

One semester of collegiate study, or the equivalent, of mathematics at or above the level of pre-calculus and/or probability, statistics, and data analysis is required for the General Education Program. Students should consult their appointed University advisor in determining the requirements for their anticipated degree.

Course Objective: To provide the student with an introduction to the basic concepts of the course (including, but not limited to those outlined in the bulletin description), as well as any applications that may be pertinent to her/his specific area of study, eventually leading to a clear understanding and mastery of all topics and techniques covered.

Homework: Three problems sets will be assigned throughout the semester and collected every friday at a time specified by the instructor.

Quizzes: Ten to twelve quizzes will be given throughout the semester. The use of calculators will not be allowed during any quiz. The lowest quiz grade will be dropped from the final grade calculation. No unannounced quizzes will be given.

Exams: Two 40-50 minute exams will be given during the semester, as well as one cumulative exam on **Tuesday, December** 15^{th} **from 10:30am to 12:30pm**. The use of calculators will not be allowed during the exam. The instructor reserves the right to make any portion of an exam in-class or take-home.

Miscellaneous: Each student will prepare and present solutions to no more than two problems from any one section of the text. Presentations will be given during office hours and (time permitting) in class. A list of problems will be determined by the instructor.

Attendance: Not required. Regularly attending classes, however, is strongly recommended. Students are responsible for obtaining lecture notes from any classes missed. No make-up exams/quizzes will be given without proper documentation from the Office of the Vice Provost for Undergraduate Education.

Cheating will not be tolerated, and cell phones will be turned off prior to class!

Grading Breakdown:

Event	Date	Weight
Exam 1	TBD	10%
Exam 2	TBD	10%
Final Exam	December 15^{th}	20%
HW	——	45%
Quizzes	——	10%
Presentation	——	5%

There will be NO opportunities for extra credit.

Grading Scheme (used for exams and when calculating the final grade):

A	100-93	B-	82-80	D+	69-67
A-	92-90	C+	79-77	D	66-63
B+	89-87	C	76-73*	D-	62-60
B	86-83	C-	72-70	E	Below 60

*AMAT220 is A-E graded unless the student has arranged for S/U grading. A grade of S (satisfactory) is 73% or greater.

Tutoring: Tutoring for all Math/Stat general education courses is offered Monday through Friday from approximately 9am to 4pm in ES 138 (except during Summer Sessions). A list of private tutors may be obtained at the Mathematics and Statistics main office in ES 110. A link to the office of Academic Support Services is also provided on the instructor's website. Students who are struggling with the material are strongly encouraged to attend office hours and/or visit the tutoring room regularly.

B Sample Linear Algebra Exam

Name: ______________________

December 15, 2009

Be sure to show all your work (no work = no credit). The exam will be scored out of 100 points (110 possible).

I. (15 points) Choose five terms from the list below, order them beginning with the term that you understand best, and state their definitions using the spaces provided. Included among your five choices MUST be the following terms: (1) β-coordinate Vector of $\mathbf{x}$ and (2) Subspace of a vector space. These two terms may be placed anywhere in your ordering.

Subspace	Linear Transformation	Eigenvector
Basis	Linearly Dependent Set	Matrix Product AB
Span	β-coordinate Vector of $\mathbf{x}$	

a. (5 pts)

b. (4 pts)

c. (3 pts)

d. (2 pts)

e. (1 pt)

II. (10 pts) Let A be an $m \times n$ matrix. Either circle or fill in the blank with the correct answer for each of the following statements.

a. Col A/Nul A is a subspace of $\mathbb{R}^m$.

b. (Row $A)^{\perp}$ =Nul A/Nul A^T.

c. If $\mathbf{v}$ is in Col A, then the equation ________________________ is consistent.

d. The standard basis for $\mathbb{P}_n$ is ________________________.

e. A linear transformation $T : V \to W$ is said to be ________________________ if each $\mathbf{b}$ in W is the image of at least one $\mathbf{x}$ in V.

III. (13 points) Provide a complete proof of the following theorem.

Theorem: If a vector space V has a basis $\beta = \{\mathbf{b_1}, \ldots, \mathbf{b_n}\}$, then any set in V containing more than n vectors must be a linearly dependent set.

IV. (7 pts) Provide a complete proof of the following theorem.

Theorem: If $S = \{\mathbf{u_1}, \ldots, \mathbf{u_p}\}$ is an orthogonal set of nonzero vectors in $\mathbb{R}^n$, then S is linearly independent and hence a basis for the subspace spanned by S.

V. (10 pts) Let $\beta = \{\begin{bmatrix} 1/\sqrt{5} \\ -2/\sqrt{5} \end{bmatrix}, \begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}\}$. Prove that β is an orthonormal basis for $\mathbb{R}^2$ and find the β-coordinates of $\mathbf{y} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$.

VI. (32 pts) Complete the following for the given matrix.

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

a. (5 pts) Find the characteristic polynomial of A. You need not simplify your answer.

b. (8 pts) $\lambda = -1$ is an eigenvalue of A. Find a basis for the eigenspace of A corresponding to $\lambda = -1$.

c. (4 pts) How many remaining eigenvalues of A are there? Find them.

d. (6 pts) Diagonalize A, if possible.

e. (5 pts) Orthogonally diagonalize A, if possible.

f. (4 pts) Without performing a cofactor expansion, find the determinant of A.

VII. (5 pts) Use the properties of transposes to prove the following fact.

If a matrix A is orthogonally diagonalizable, then it is symmetric.

VIII. (18 pts)

a. (13 pts) Let H denote the set of all polynomials of degree at most two, the sum of whose coefficients is zero, i.e.,

$$H = \{p(t) = a_2t^2 + a_1t + a_0 | a_2 + a_1 + a_0 = 0\}.$$

Prove that H is a subspace of $\mathbb{P}_2$.

b. (5 pts) Let K denote the set of all polynomials of degree at most one, the product of whose coefficients is zero, i.e.,

$$K = \{p(t) = a_1t + a_0 | a_1a_0 = 0\}.$$

Show that K is not closed under vector addition, and consequently is <u>not</u> a subspace of $\mathbb{P}_1$.

C Sample Calculus Exam

Name: ____________________________

AMAT 113 Section 13356

March 25, 2010

Be sure to show all your work (no work = no credit).

I. 12 points

1. (3 pts.) Consider the sequence $\{r^n\}$, where r is a real number. For what values of r is the sequence convergent, and in each case, what does it converge to?
2. (4 pts.) Now consider the series $\sum_{n=1}^{\infty} r^n$. For what values of r is the series convergent, and in each case, what is its sum?
3. (5 pts.) What is wrong with the following equation?
   Hint: Use your answer in part (2).

$$\sum_{n=1}^{\infty} r^n = -\frac{1}{2}$$

II. 34 points

Match each of the given series with the appropriate test from those listed (2 pts each). Then chose TWO series and determine whether they converge or diverge (14 pts each). You may only use each test once!

$\sum_{n=0}^{\infty} \frac{e^n}{(\sqrt{2})^n + \pi^n}$	Root Test
	Alternating Series Test
$\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln(n^2)}$	Integral Test
	Comparison Test
$\sum_{n=0}^{\infty} \frac{(-1)^n n}{2n^2 + 1}$	Test for Divergence

III. 16 points

Let $a_n = \dfrac{\sin(\frac{n\pi}{2})}{1+n}$. Answer the following.

1. (2 pts.) List the first 4 values of the sequence $\{a_n\}$, starting with $n = 0$.
2. (2 pts.) Is $\{a_n\}$ bounded above? Is it bounded below? If yes, determine bound(s) of the sequence. Justify your answers.
3. (2 pts.) Does the sequence converge or diverge? If it is convergent, determine the value L that it converges to.
4. (5 pts.) Consider the series $\sum_{n=0}^{\infty} \frac{1}{(1+n)^n}$. Use the Root Test to show whether or not the given series converges.
5. (5 pts.) What can you conclude about the series $\sum_{n=0}^{\infty} (a_n)^n$? Justify your answer using a theorem from the text.

IV. 14 points

Given the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, where $p \in \mathbb{R}$, answer the following.

1. (2 pts.) When does the given series converge? When does it diverge?
2. (4 pts.) If $f(x) = \frac{1}{x^p}$, for $p \neq 1$, complete the following equation.

$$\int_1^{\infty} f(x)dx = \lim_{t\to\infty} [\underline{\hspace{5cm}}]$$

3. (4 pts.) Determine when the limit in part (2) converges and when it diverges. Justify your answer.
4. (4 pts.) Provide <u>two</u> graphs that illustrate the correlation between the function $f(x) = \frac{1}{x^p}$ over the interval $[1, \infty)$ and the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$. One graph should show convergence of the series, and one divergence.