

input A, B, C

Sort A, B, C into $R1 \leq R2 \leq R3$

$\{true\}$

if $(A > B)$

then $temp \leftarrow A$

$A \leftarrow B$

$B \leftarrow temp$

if $(B \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow B$

$R3 \leftarrow C$

else $R3 \leftarrow B$

if $(A \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow C$

else $R1 \leftarrow C$

$R2 \leftarrow A$

$\{R1 \leq R2 \leq R3\}$

input A, B, C

$\{true\}$

if $(A > B)$

then $temp \leftarrow A$

$A \leftarrow B$

$B \leftarrow temp$

$\{A \leq B\}$

Here we want A and B ordered

if $(B \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow B$

$R3 \leftarrow C$

else $R3 \leftarrow B$

if $(A \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow C$

else $R1 \leftarrow C$

$R2 \leftarrow A$

$\{R1 \leq R2 \leq R3\}$

input A, B, C

$\{true\}$

if $(A > B)$

$\neg(A > B) \Rightarrow (A \leq B)$ (*test false*)

then $temp \leftarrow A$

$A \leftarrow B$

$B \leftarrow temp$

$\{A \leq B\}$

Here we want A and B ordered

if $(B \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow B$

$R3 \leftarrow C$

else $R3 \leftarrow B$

if $(A \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow C$

else $R1 \leftarrow C$

$R2 \leftarrow A$

$\{R1 \leq R2 \leq R3\}$

input A, B, C

$\{true\}$

if $(A > B)$

$\neg(A > B) \Rightarrow (A \leq B)$ (*test false*)

$\{B \leq A\}$

(*test true*) $\Rightarrow \{B \leq A\}$

then $temp \leftarrow A$

$\{B \leq temp\}$

$A \leftarrow B$

$\{A \leq temp\}$

$B \leftarrow temp$

$\{A \leq B\}$

So we have A and B ordered

if $(B \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow B$

$R3 \leftarrow C$

else $R3 \leftarrow B$

if $(A \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow C$

else $R1 \leftarrow C$

$R2 \leftarrow A$

$\{R1 \leq R2 \leq R3\}$

$\{A \leq B\}$
if $(B \leq C)$

Here we have A and B ordered

then $R1 \leftarrow A$

$R2 \leftarrow B$

$R3 \leftarrow C$

else $R3 \leftarrow B$

if $(A \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow C$

else $R1 \leftarrow C$

$R2 \leftarrow A$

$\{R1 \leq R2 \leq R3\}$

This is what we want

$\{A \leq B\}$ Here we have A and B ordered
if $(B \leq C)$
 $\{A \leq B\} \wedge \{(B \leq C)(\text{test true})\} \Rightarrow \{A \leq B \leq C\}$
then $R1 \leftarrow A$

$R2 \leftarrow B$

$R3 \leftarrow C$

$\{R1 \leq R2 \leq R3\}$ This is what we want

else $R3 \leftarrow B$

if $(A \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow C$

else $R1 \leftarrow C$

$R2 \leftarrow A$

$\{R1 \leq R2 \leq R3\}$ This is what we want

$\{A \leq B\}$	Here we have A and B ordered
<i>if</i> $(B \leq C)$	
$\{A \leq B\} \wedge \{(B \leq C)(\text{test true})\} \Rightarrow \{A \leq B \leq C\}$	
<i>then</i> $R1 \leftarrow A$	
	$\{R1 \leq B \leq C\}$
$R2 \leftarrow B$	
	$\{R1 \leq R2 \leq C\}$
$R3 \leftarrow C$	
	$\{R1 \leq R2 \leq R3\}$

else $R3 \leftarrow B$

if $(A \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow C$

else $R1 \leftarrow C$

$R2 \leftarrow A$

$\{R1 \leq R2 \leq R3\}$

This is what we want

$\{A \leq B\}$ <i>if</i> $(B \leq C)$ $\{A \leq B\} \wedge \{B \leq C\}$ (<i>test true</i>) <i>then</i> $R1 \leftarrow A$ $R2 \leftarrow B$ $R3 \leftarrow C$	Here we have A and B ordered $\Rightarrow \{A \leq B \leq C\}$ $\{R1 \leq B \leq C\}$ $\{R1 \leq R2 \leq C\}$ $\{R1 \leq R2 \leq R3\}$
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$\{A \leq B\} \wedge \{B > C\}$ (<i>test false</i>) <i>else</i> $R3 \leftarrow B$	$\Rightarrow \{A \leq B\} \wedge \{C \leq B\}$
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if $(A \leq C)$

then $R1 \leftarrow A$

$R2 \leftarrow C$

else $R1 \leftarrow C$

$R2 \leftarrow A$

$\{R1 \leq R2 \leq R3\}$	This is what we want
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$\{A \leq B\}$	Here we have A and B ordered
<i>if</i> $(B \leq C)$	
$\{A \leq B\} \wedge \{B \leq C\}$ (<i>test true</i>)	$\Rightarrow \{A \leq B \leq C\}$
<i>then</i> $R1 \leftarrow A$	
	$\{R1 \leq B \leq C\}$
$R2 \leftarrow B$	
	$\{R1 \leq R2 \leq C\}$
$R3 \leftarrow C$	
	$\{R1 \leq R2 \leq R3\}$
<hr/>	
$\{A \leq B\} \wedge \{B > C\}$ (<i>test false</i>)	$\Rightarrow \{A \leq B\} \wedge \{C \leq B\}$
<i>else</i> $R3 \leftarrow B$	
	$\{A \leq R3\} \wedge \{C \leq R3\}$
<i>if</i> $(A \leq C)$	
<i>then</i> $R1 \leftarrow A$	
	$R2 \leftarrow C$
<hr/>	
<i>else</i> $R1 \leftarrow C$	
	$R2 \leftarrow A$
<hr/>	
$\{R1 \leq R2 \leq R3\}$	This is what we want

$\{A \leq B\}$	Here we have A and B ordered
<i>if</i> $(B \leq C)$	
$\{A \leq B\} \wedge \{B \leq C\}$ (<i>test true</i>)	$\Rightarrow \{A \leq B \leq C\}$
<i>then</i> $R1 \leftarrow A$	
	$\{R1 \leq B \leq C\}$
$R2 \leftarrow B$	
	$\{R1 \leq R2 \leq C\}$
$R3 \leftarrow C$	
	$\{R1 \leq R2 \leq R3\}$
<hr/>	
$\{A \leq B\} \wedge \{B > C\}$ (<i>test false</i>)	$\Rightarrow \{A \leq B\} \wedge \{C \leq B\}$
<i>else</i> $R3 \leftarrow B$	
	$\{A \leq R3\} \wedge \{C \leq R3\}$
<i>if</i> $(A \leq C)$	
$\{A \leq R3\} \wedge \{C \leq R3\} \wedge \{(A \leq C)\}$ (<i>test true</i>)	
<i>then</i> $R1 \leftarrow A$	
	$R2 \leftarrow C$
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<i>else</i> $R1 \leftarrow C$	
	$R2 \leftarrow A$
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$\{R1 \leq R2 \leq R3\}$	This is what we want

$\{A \leq B\}$ <i>if</i> $(B \leq C)$ $\{A \leq B\} \wedge \{B \leq C\}$ (<i>test true</i>) <i>then</i> $R1 \leftarrow A$ $R2 \leftarrow B$ $R3 \leftarrow C$	Here we have A and B ordered $\Rightarrow \{A \leq B \leq C\}$ $\{R1 \leq B \leq C\}$ $\{R1 \leq R2 \leq C\}$ $\{R1 \leq R2 \leq R3\}$
$\{A \leq B\} \wedge \{B > C\}$ (<i>test false</i>) <i>else</i> $R3 \leftarrow B$ <i>if</i> $(A \leq C)$ $\{A \leq R3\} \wedge \{C \leq R3\} \wedge \{(A \leq C)\}$ <i>then</i> $R1 \leftarrow A$ $R2 \leftarrow C$	$\Rightarrow \{A \leq B\} \wedge \{C \leq B\}$ $\underline{\{A \leq R3\} \wedge \{C \leq R3\}}$ $\Rightarrow \{(A \leq C \leq R3)\}$
<i>else</i> $R1 \leftarrow C$ $R2 \leftarrow A$	
$\{R1 \leq R2 \leq R3\}$	This is what we want

$\{A \leq B\}$ <i>if</i> $(B \leq C)$ $\{A \leq B\} \wedge \{B \leq C\}$ (<i>test true</i>) <i>then</i> $R1 \leftarrow A$ $R2 \leftarrow B$ $R3 \leftarrow C$	Here we have A and B ordered $\Rightarrow \{A \leq B \leq C\}$ $\{R1 \leq B \leq C\}$ $\{R1 \leq R2 \leq C\}$ $\{R1 \leq R2 \leq R3\}$
$\{A \leq B\} \wedge \{B > C\}$ (<i>test false</i>) <i>else</i> $R3 \leftarrow B$ <i>if</i> $(A \leq C)$ $\{A \leq R3\} \wedge \{C \leq R3\} \wedge \{(A \leq C)\}$ <i>then</i> $R1 \leftarrow A$ $R2 \leftarrow C$	$\Rightarrow \{A \leq B\} \wedge \{C \leq B\}$ $\underline{\{A \leq R3\} \wedge \{C \leq R3\}}$ $\Rightarrow \{(A \leq C \leq R3)\}$ $\{R1 \leq C \leq R3\}$ $\{R1 \leq R2 \leq R3\}$
<i>else</i> $R1 \leftarrow C$ $R2 \leftarrow A$	
$\{R1 \leq R2 \leq R3\}$	This is what we want

$\{A \leq B\}$	Here we have A and B ordered
<i>if</i> $(B \leq C)$	
$\{A \leq B\} \wedge \{B \leq C\}$ (<i>test true</i>)	$\Rightarrow \{A \leq B \leq C\}$
<i>then</i> $R1 \leftarrow A$	
	$\{R1 \leq B \leq C\}$
$R2 \leftarrow B$	
	$\{R1 \leq R2 \leq C\}$
$R3 \leftarrow C$	
	$\{R1 \leq R2 \leq R3\}$
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$\{A \leq B\} \wedge \{B > C\}$ (<i>test false</i>)	$\Rightarrow \{A \leq B\} \wedge \{C \leq B\}$
<i>else</i> $R3 \leftarrow B$	
	$\{A \leq R3\} \wedge \{C \leq R3\}$
<i>if</i> $(A \leq C)$	
$\{A \leq R3\} \wedge \{C \leq R3\} \wedge \{(A \leq C)\}$	$\Rightarrow \{(A \leq C \leq R3)\}$
<i>then</i> $R1 \leftarrow A$	
	$\{R1 \leq C \leq R3\}$
$R2 \leftarrow C$	
	$\{R1 \leq R2 \leq R3\}$
<hr/>	
$\{A \leq R3\} \wedge \{C \leq R3\} \wedge \{(A > C)\}$ (<i>test false</i>)	
<i>else</i> $R1 \leftarrow C$	
$R2 \leftarrow A$	
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$\{R1 \leq R2 \leq R3\}$	This is what we want

$\{A \leq B\}$	Here we have A and B ordered
<i>if</i> $(B \leq C)$	
$\{A \leq B\} \wedge \{B \leq C\}$ (<i>test true</i>)	$\Rightarrow \{A \leq B \leq C\}$
<i>then</i> $R1 \leftarrow A$	
	$\{R1 \leq B \leq C\}$
$R2 \leftarrow B$	
	$\{R1 \leq R2 \leq C\}$
$R3 \leftarrow C$	
	$\{R1 \leq R2 \leq R3\}$
<hr/>	
$\{A \leq B\} \wedge \{B > C\}$ (<i>test false</i>)	$\Rightarrow \{A \leq B\} \wedge \{C \leq B\}$
<i>else</i> $R3 \leftarrow B$	
	$\{A \leq R3\} \wedge \{C \leq R3\}$
<i>if</i> $(A \leq C)$	
$\{A \leq R3\} \wedge \{C \leq R3\} \wedge \{(A \leq C)\}$	$\Rightarrow \{(A \leq C \leq R3)\}$
<i>then</i> $R1 \leftarrow A$	
	$\{R1 \leq C \leq R3\}$
$R2 \leftarrow C$	
	$\{R1 \leq R2 \leq R3\}$
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$\{A \leq R3\} \wedge \{C \leq R3\} \wedge \{(A > C)\}$	$\Rightarrow \{C \leq A \leq R3\}$
<i>else</i> $R1 \leftarrow C$	
$R2 \leftarrow A$	
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$\{R1 \leq R2 \leq R3\}$	This is what we want

$\{A \leq B\}$ <i>if</i> $(B \leq C)$ $\{A \leq B\} \wedge \{B \leq C\}$ (<i>test true</i>) <i>then</i> $R1 \leftarrow A$ $R2 \leftarrow B$ $R3 \leftarrow C$	Here we have A and B ordered $\Rightarrow \{A \leq B \leq C\}$ $\{R1 \leq B \leq C\}$ $\{R1 \leq R2 \leq C\}$ $\{R1 \leq R2 \leq R3\}$
$\{A \leq B\} \wedge \{B > C\}$ (<i>test false</i>) <i>else</i> $R3 \leftarrow B$ <i>if</i> $(A \leq C)$ $\{A \leq R3\} \wedge \{C \leq R3\} \wedge \{(A \leq C)\}$ <i>then</i> $R1 \leftarrow A$ $R2 \leftarrow C$	$\Rightarrow \{A \leq B\} \wedge \{C \leq B\}$ $\underline{\{A \leq R3\} \wedge \{C \leq R3\}}$ $\Rightarrow \{(A \leq C \leq R3)\}$ $\{R1 \leq C \leq R3\}$ $\{R1 \leq R2 \leq R3\}$
$\{A \leq R3\} \wedge \{C \leq R3\} \wedge \{(A > C)\}$ <i>else</i> $R1 \leftarrow C$ $R2 \leftarrow A$	$\Rightarrow \{C \leq A \leq R3\}$ $\{R1 \leq A \leq R3\}$ $\{R1 \leq R2 \leq R3\}$
$\{R1 \leq R2 \leq R3\}$	Q.E.D.