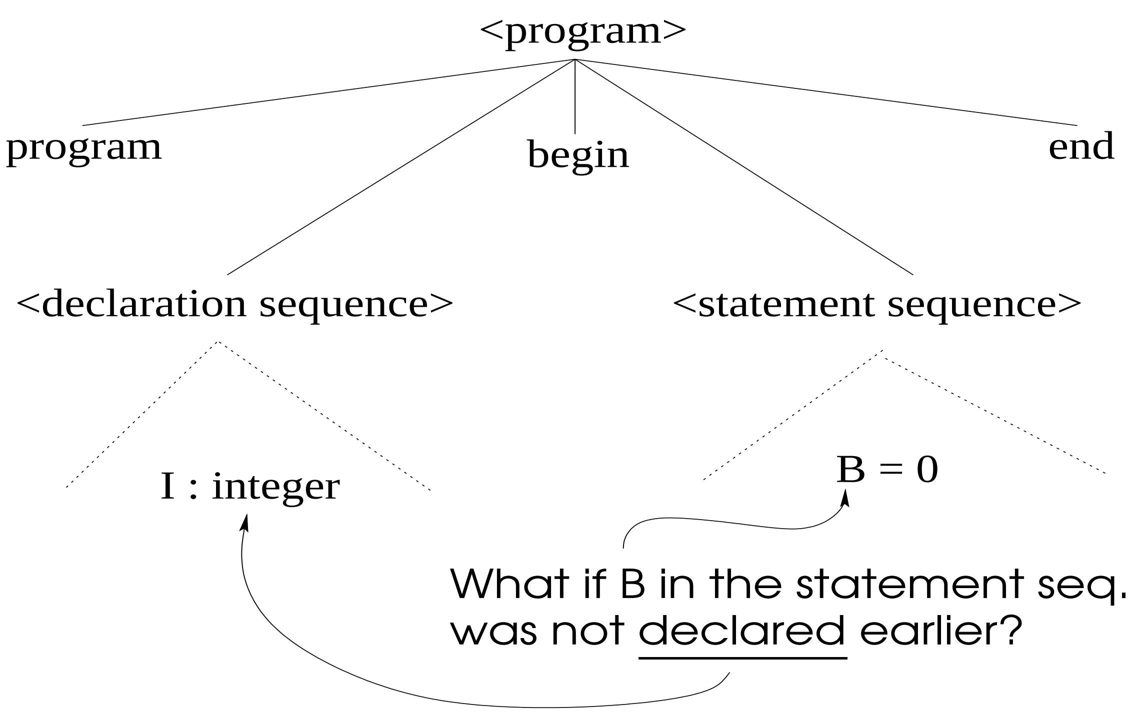
Example: Context Sensitivity

Consider the following partially developed parse tree from a BNF (context-free) grammar:



If at the point where identifier B is used, it is undeclared, then even though the various statements and declarations are themselves well-formed, the program violates the semantics of the language.

However, this is the so-called *static semantics*. In other words, the error is not really dependent upon program execution; it can be detected statically at compile time.

Goal: Relegate static semantics to parsing.

Problem: Context-free grammars cannot enforce agreement between declaration and use; ‘‘double word’’ problem.

This can be handled by context-*sensitive* grammars.

$$\langle S\rangle \rightarrow aa \mid bb \mid a\langle M\rangle\langle A_0\rangle \mid b\langle M\rangle\langle B_0\rangle$$

$$\langle M\rangle \rightarrow a\langle A\rangle \mid b\langle B\rangle \mid a\langle M\rangle\langle A\rangle \mid b\langle M\rangle\langle B\rangle$$

(so far, these rules are context-free, but...)

$$\langle A\rangle a \rightarrow a\langle A\rangle \qquad \langle A\rangle\langle A_0\rangle \rightarrow a\langle A_0\rangle$$

$$\langle A\rangle b \rightarrow b\langle A\rangle \qquad \langle B\rangle\langle A_0\rangle \rightarrow a\langle B_0\rangle$$

$$\langle B\rangle a \rightarrow a\langle B\rangle \qquad \langle A\rangle\langle B_0\rangle \rightarrow b\langle A_0\rangle$$

$$\langle B\rangle b \rightarrow b\langle B\rangle \qquad \langle B\rangle\langle B_0\rangle \rightarrow b\langle B_0\rangle$$

$$\langle A_0\rangle \rightarrow a$$

$$\langle B_0\rangle \rightarrow b$$

This context-sensitive grammar generates all sentences W in $\{a,b\}^+$ of the form XX.

$$<S> \;\to\; aa \mid bb \mid a<M><A_0> \mid b<M><B_0>$$
$$<M> \;\to\; a<A> \mid b<B> \mid a<M><A> \mid b<M><B>$$

$$<A>a \;\to\; a<A> \qquad\qquad <A><A_0> \;\to\; a<A_0>$$
$$<A>b \;\to\; b<A> \qquad\qquad <B><A_0> \;\to\; a<B_0>$$
$$<B>a \;\to\; a<B> \qquad\qquad <A><B_0> \;\to\; b<A_0>$$
$$<B>b \;\to\; b<B> \qquad\qquad <B><B_0> \;\to\; b<B_0>$$

$$<A_0> \;\to\; a$$
$$<B_0> \;\to\; b$$

The context-free rules generate palindrome-like sentences of the form $XX^{r\prime}$, where X^r is the reverse of X, and $X^{r\prime}$ is like X^r but with non-terminals.

Example: aabaab   First derive aab<B><A><A^0>.

$<S>$

$a <M> <A_0>$

$a\ a <M> <A> <A_0>$

$a\ a\ b <B> <A> <A_0>$

The zero-subscripted non-terminals always
mark the right end of the derived sequence.

$$\langle S\rangle \rightarrow aa \mid bb \mid a\langle M\rangle\langle A_0\rangle \mid b\langle M\rangle\langle B_0\rangle$$
$$\langle M\rangle \rightarrow a\langle A\rangle \mid b\langle B\rangle \mid a\langle M\rangle\langle A\rangle \mid b\langle M\rangle\langle B\rangle$$

$$\langle A\rangle a \rightarrow a\langle A\rangle \qquad \langle A\rangle\langle A_0\rangle \rightarrow a\langle A_0\rangle$$
$$\langle A\rangle b \rightarrow b\langle A\rangle \qquad \langle B\rangle\langle A_0\rangle \rightarrow a\langle B_0\rangle$$
$$\langle B\rangle a \rightarrow a\langle B\rangle \qquad \langle A\rangle\langle B_0\rangle \rightarrow b\langle A_0\rangle$$
$$\langle B\rangle b \rightarrow b\langle B\rangle \qquad \langle B\rangle\langle B_0\rangle \rightarrow b\langle B_0\rangle$$

$$\langle A_0\rangle \rightarrow a$$
$$\langle B_0\rangle \rightarrow b$$

$a\ a\ b\ \langle B\rangle\ \langle A\rangle\ \langle A_0\rangle$

The zero-subscripted non-terminals get converted to terminals and then exchanged with ordinary non-terminals to the left.

$a\ a\ b\ \langle B\rangle\ a\ \langle A_0\rangle$

Terminals exchange with ordinary non-terminals to their left.

$a\ a\ b\ a\ \langle B\rangle\ \langle A_0\rangle$

Now convert $\langle A_0\rangle$ and switch with $\langle B\rangle$.

$a\ a\ b\ a\ a\ \langle B_0\rangle$

Finally, convert $\langle B_0\rangle$ to terminal.

a a b a a b