Name: SOLUTIONS

Physics 240, Exam #2
Monday, October 24, 2016 (2:45-4:05 p.m.)

Instructions: Complete all questions as best you can and show all of your work. If you just write down an answer without explaining how you got it, you will not receive any points.

Do not use any device to retrieve text, equations, or algorithms during the exam. You can use a scientific calculator but no tablets nor smartphones. Circle or box your final answers.

You may also use the one 2-sided formula sheet provided. Don't consult with or discuss this exam with neighbors. Good luck, and budget enough time. Read all of the problems first.

Use the space below if you run out of room to answer the questions on any upcoming sheet. You can also use the back side of each problem and/or request additional sheets from me.

TOTAL POSSIBLE POINTS = 50 / 50 [10 points summed per problem]
1. CONCEPTUAL QUESTIONS (Briefly justify answers, mentioning equations if need be.)

10 points total

(a.) Based on formulae you know for special relativity, name and explain at least one key feature of a hypothetical tachyon particle, which if exists travels faster than light. (2 points)

(b.) What is the sure-fire, but 1-way-trip-ticket, way to “time travel” to the future, based on what you learned from Einstein, but is this even feasible with current technology? (2 points)

(c.) Name and explain 1 example from an experimental result or discovery from the 19th or 20th century hinting that the classical, Newtonian picture of nature is incomplete. (2 points)

(d.) In terms of energy and our modern-day understanding of the nature of the photon, the particle of light, explain what amplitude, frequency, and wavelength mean for it. (2 points)

(e.) Name two examples of (approximate) blackbody radiators (discussed in class, or your own) and explain how it is possible to be one without literally being black in color (2 points)

(a.) In order to travel faster than the speed of light tachyons would have to have a negative squared-mass, necessitating an imaginary or complex rest mass (meaning unknown, and never at “rest” anyway, not possible) and they would appear to propagate “backwards through time” relative to the outside universe. They can never slow down to the speed of light or lower, requiring an infinite amount of energy to do so, and they speed up when they lose energy instead of slowing down. For more see Conceptual Example 2.12 on pp. 67-68 (Can also say that beta is greater than 1, making gamma a complex number.)

(b.) Go close to speed of light and return. Not feasible now because our rockets don’t go anywhere near as fast as necessary due to fuel and force requirements, but not technically against any laws of physics.

(c.) Potential examples include the photoelectric effect, blackbody radiation and the UV catastrophe, spectral lines, cathode rays, x-rays, etc. (consult your lecture notes as well as Chapters 3 and 4 for even more, with their respective explanations).

(d.) Amplitude is meaningless for a single photon, but for a source of many (like a light bulb or laser) it can be used to signify the number of photons. It has nothing to do with energy for a single photon, even though it increases with energy for a traditional wave, but can stand for total energy adding over many photons each of fixed tiny energy depending on the units used. Frequency and wavelength are inversely proportional to each other, with the constant of proportionality being the speed of light. E = hf = hc / lambda, and so a higher frequency means more energy, and lower wavelength means more energy, and vice versa.

(e.) The Sun and other stars, a human body, the universe itself (Cosmic Microwave Background radiation), a light bulb, an oven, the cavity example, anything hot basically radiating heat and/or light, the asphalt pavement of the road on a hot day, a candle; you can be a blackbody just approximately, having a spectrum of output wavelengths with a peak somewhere. Also, you can still absorb and re-emit “colors” of light other than the visible. You can have an emissivity somewhere less than 1, but still follow the same power law of emission -- lots of different possible answers, all equally correct.
2. Plans are laid out for exploring the center of the galaxy. This is approximately 26,000 light years from earth.

10 points total

(a.) What speed (relative to the Earth frame) would a starship have to achieve in order to reach the center of the Milky Way during one human lifetime, as experienced by the occupants of that outbound spacecraft, despite Earth experiencing 26,000 years approximately during the same frame of time? Note: You may quantitatively interpret how many years “one human lifetime” means in any reasonably justifiable way. (4 points)

(b.) What would the apparent distance be as a result of length contraction? (2 points)

(c.) For a rocket with a rest mass of 200 million kg, what would its kinetic energy, the total energy, and the momentum be at the speed from part (a.)? (3 points)

(d.) Assuming technology continues to improve, could this voyage be possible? (1 point)

In order to get the right answer most easily for this one you must quote actual velocity [m/s] not just beta, and must use the more precise value of the speed of light c provided in formula sheet.

(a.) This is essentially identical to Example 2.2 pp. 34-35 of Rex and Thornton, which we also went over in class, where (skipping a few steps here on how to get there) the equation

\[ T = \frac{L}{v} = \frac{T_0'}{\sqrt{1 - \frac{v^2}{c^2}}} \]

is used. (No factor of 2 here because I said nothing about a return trip.) \( L = 26,000 \) light-years, which converted into meters is \( \sim 2.460 \times 10^{20} \) meters. (1 light-year = distance that light travels in one year in the vacuum of space, going at \( c = 299,792,458 \) m/s, so you just do the appropriate conversions, or look at formula sheet for answer.) We are solving for ‘v.’ \( T_0' \) is a matter of interpretation, you just need to choose something reasonable. Let’s say 50 years (starting at age 20 and living until 70). 50 years \( \sim 1.578 \times 10^9 \) seconds.

The answer (using 50 years or something close to that) is \( v = 299,791,903.652 \) m/s, extremely close to light speed. (This is a beta of 0.99999815) Note how the occupants of the ship will still be alive despite thousands of years having passed upon the Earth -- isn’t it incredible?!

Alternatively, because one can surmise that the speed must be very close to the speed of light before even starting the problem, and because you are given that “approximately 26,000 years have passed on Earth” during the voyage, then for full credit you can instead do \( T = \gamma T_0 \), where \( \gamma = \frac{T}{T_0} \sim 26,000 \text{ years} / 50 \text{ years} = 520 \). Then solve for beta using the equation \( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \). Beta is 0.99999815, same as above.

(b.) \( L = \frac{L_0}{\gamma} = 26,000 \) light-years / 520 = 50 light-years OR 4.730 x 10^17 meters

(c.) \( KE = (\gamma - 1) \cdot mc^2 = 9.329 \times 10^7 \) Joules
Total Energy = \( \gamma m c^2 = 9.347 \times 10^7 \) Joules
Momentum = \( \gamma m v = 3.118 \times 10^{19} \) kg \( \times \) m/s
(Many alternative ways to do part c. as well – see your equation sheet.)

(d.) Yes, it is improbable but NOT impossible, as this would not break any laws of physics.
3. Mary is traveling on a spaceship away from her twin brother Frank, who remains on Earth. Frank is sending out light signals to Mary during both the outbound and return trips. During the outbound trip the source (Frank) and receiver (Mary) are receding. For the return trip, Mary’s direction is of course reversed, and her absolute speed is greater. Frank sends signals of frequency $f_0$, but Mary receives them at a different frequency, $f = (1/3) \times f_0$ because of the Doppler effect. But, during her return trip, she receives then at frequency $= 4 \times f_0$ instead.

10 points total

(a.) What were Mary’s outbound and inbound velocities, in terms of beta (explicit values in m/s not needed just $v =$ something times beta) (4 points)

(b.) Find the relationship between $\lambda$ and $\lambda_0$, which are the respective wavelengths of light corresponding to $f$ and $f_0$ (1 point)

(c.) What is the difference in energy of the photons between source and receiver during both legs of the trip? What does this say about “conservation of energy”? (3 points)

(d.) If Frank used yellow light, then on the outbound trip would Mary be more likely to need UV or IR glasses? What about for observing the light on her return trip? (2 points)

(a.) $f / f_0 = 1/3 = \sqrt{(1 + \beta)} / \sqrt{(1 - \beta)}$ for source and receiver separating. Square both sides and solve for beta, $\beta = 0.8$, or a velocity of $0.8c \sim 240$ million m/s (‘-‘ means receding). $f / f_0 = 4 = \sqrt{(1 + \beta)} / \sqrt{(1 - \beta)}$ for source and receiver coming together. Square both sides and solve for beta, $\beta = 0.8824$, or a velocity of that times c, equal to $264.5$ million m/s

(b.) $f = f_0 / 3$ (outbound) => $\lambda = \lambda_0 \times 3$ and $f = f_0 \times 4$ (inbound) => $\lambda = \lambda_0 / 4$ (since $c = \lambda \times f$)

(c.) $E = h \times f$, so energy divided by 3 outbound, and multiplied by 4 inbound. Conservation of energy works well when defined amongst particles within a particular reference frame only, and not across frames. Also accepted: it’s wrong (classical) or it’s only approximate.

(d.) When Mary is receding the light would be redshifted, so the yellow would appear as IR. On the return leg, the light is blue-shifted (greater frequency, but shorter wavelength and thus higher in energy) so the answer is UV is seen (closer to blue on the rainbow). Tripling or quadrupling wavelength or dividing by 3 or 4 can’t stay within visible light: 400 - 700 nm
4. Light of wavelength 400 nm is incident upon lithium (work function $\phi$, Greek letter phi, equal to 2.93 eV in energy).

10 points total

(a.) Calculate the energy of just one photon in that beam of light. What color is this light? (2 points) *Credit for being close on color*

(b.) What is the retarding potential $V_0$ that would stop all photoelectrons? (3 points) *Hint: solve for $0$ kinetic energy*

(c.) The source of light is changed. What frequency of light is needed to produce electrons of kinetic energy 3 eV from illumination of lithium? (3 points)

(d.) Find the wavelength of the light in (c.) and discuss where it is in the EM spectrum (or, in other words, is it visible, UV, IR, radio, microwaves, etc.?) (2 points)

Example 3.11 (a) (b) is (a) (b) here and 3.12 (a) (b) is (c) (d) here. Section 3.6 p. 109.

(a.) $3.10 \text{ eV} \approx 5 \times 10^{-19} \text{ J}; \text{ Color: full credit for saying Blue, Indigo, Violet, or UV}$

(b.) 0.17 V

(c.) $1.43 \times 10^{15} \text{ Hz}$

(d.) 210 nm (UV)
5. An x-ray photon of wavelength 0.05 nm scatters off of a gold target.

(a.) Can the x-ray be Compton-scattered from an electron bound by as much as 62 keV? Why or why not? Assume the outermost electrons of gold can be approximated as free ones, but that here we mean an inner one. (2 points)

(b.) What is the longest wavelength of scattered photon that may be observed? (3 points) Now assume we are dealing with quasi-free outer-shell electrons.

(c.) What is the kinetic energy of the most energetic recoil electron and at what angle does it occur? (3 points)

(d.) Is it possible to determine the wavelength and energy of the incident photon without measuring them directly from the said photon? If so, how? (2 points)

Note you don’t need to know anything additional about gold (mass, etc.) to solve this problem.

(a.) is Example 3.16. Answer is no because the x-ray energy is only 24.8 keV, so that’s not enough to dislodge the electron.

(b.) 3.16 part B. Answer is 0.055 nm

(c.) 3.16 part C. Answer is 22.5 keV and phi = 0 degrees and theta = 180 degrees (you could say either angle for full credit)

(d.) Yes: because $\text{deltalambda does not depend on lambda or lambda-prime}$, one may determine the wavelength (and energy, using $E = \hbar c / \lambda$) of the incident photon by merely observing the kinetic energy of the electron at forward angles (see Problem 60). You can also observe the outgoing photon. There are many different ways, because there is a fixed formula we derive for Compton scattering that we only need all but one input for. You can also answer this question arguing from conservation of energy (same in principle).