DEPARTMENT OF PHYSICS

University at Albany
State University of New York

Comprehensive Field Examination

Quantum

Wednesday, May 18, 2016
10:00 AM - 1:00 PM

Instruction:

Answer any four out of five questions

Please check below the four problems you have done. Do not place your name on the examination booklets or this cover sheet. Each problem SHOULD be done in a separate examination book. The problems within each area carry equal weight.

Turn in the cover sheet and the four books at the end of the exam.

6. ___________

7. ___________

8. ___________

9. ___________

10. ___________

Student Identification Code: ________________________________

NOTE: The same code is to be used on all sections of the Comprehensive Examination taken in May 2016.
Problem 6

(a) A particle of mass $m$ is initially in the ground state of an infinite one-dimensional potential well with walls at $x = 0$ and $x = a$. If the wall is moved suddenly to $x = 8a$, calculate the approximate probabilities of finding the particle in the ground-state and first excited state of the new potential well. What inequality must hold for the approximation to be valid?

(b) A particle of mass $m$ is initially in the ground state of an infinite one-dimensional potential well with walls at $x = 0$ and $x = a$. If the wall is moved very slowly to $x = 8a$, calculate the approximate energy and wavefunction of the particle in the new well. Approximately how much work was done in the process? What inequality must hold for the approximation to be valid?
Problem 7

(a) List the ground-state electron configurations for the following atoms:
   C (Z = 6)
   Na (Z = 11)
   Ni (Z = 28)
   Nb (Z = 41) with only one electron in 5s

(b) Describe the three Hund’s rules governing the quantum numbers $S$, $L$ and $J$ of a multiple-electron atom in the ground state.

(c) Find the $S$, $L$ and $J$ quantum numbers for the above atoms using the Hund’s rules.

(d) What transition would give rise to the lowest frequency line in the absorption spectrum of Na? This frequency line is a doublet. Describe the mechanism that splits the energy levels.
Problem 8

(a) Starting with an asymptotic expression for the wavefunction at large values of \( r \),

\[ \psi \approx e^{ikz} + \frac{e^{ikr}}{r} f(\theta), \]

determine the differential and total cross–sections. Here \( \theta \) is the scattering angle.

(b) Write the expression for \( f(\theta) \) in the Born approximation. You may leave out an overall constant factor.

(c) Calculate the differential cross–section in the Born approximation for scattering from a spherical potential well:

\[ V(r) = \begin{cases} -V_0, & r < a, \\ 0, & r > a \end{cases} \]

(d) Starting from expansions

\[ \psi \approx \frac{1}{2ikr} \sum_{l=0}^{\infty} (2l + 1) P_l(\cos \theta) \left[ (-1)^{l+1} e^{-ikr} + e^{2i\delta_l} e^{ikr} \right] \]

\[ e^{ikz} \approx \frac{1}{2ikr} \sum_{l=0}^{\infty} (2l + 1) P_l(\cos \theta) \left[ (-1)^{l+1} e^{-ikr} + e^{ikr} \right], \]

determine the total cross–section in terms of the phase shifts \( \delta_l \).
Consider a two-dimensional harmonic oscillator governed by the Hamiltonian

\[ \hat{H} = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2} [x^2 + y^2] \]

(a) Write the operator of angular momentum for this system and demonstrate that it is conserved using the Heisenberg equation.

(b) Conservation of angular momentum follows from a certain symmetry, identify the transformations of coordinates and momenta under this symmetry.

(c) Determine the energies and degeneracies of the two lowest energy levels and write the corresponding wavefunctions in the basis of occupation numbers \(|n_x, n_y\).

(d) Find the leading correction to the energy of the first excited state caused by a perturbation \(V = \lambda(x + y)^2\).
Problem 10

Consider a particle moving in a periodic potential consisting of a series of evenly spaced delta function spikes in one dimension:

\[ V(x) = \alpha \sum_{j=-\infty}^{\infty} \delta(x - ja). \]

(a) Since the potential has a periodicity \( a \), describe the relationship between the wave functions \( \Psi(x) \) and \( \Psi(x + a) \), according to the Blochs theorem. The relationship involves a real parameter \( K \).

(b) Solve the Schrödinger equation and find the general solution \( \Psi(x) \) in the region \( 0 < x < a \).

(c) Using the relationship in (a), find the wave function \( \Psi(x) \) in the region \( -a < x < 0 \).

(d) From the boundary conditions at \( x = 0 \), where a delta function spike is located, derive the transcendental equation relating the particles wave vector \( k \) and the parameter \( K \). Describe how forbidden energy gap forms when \( k = n\pi/a \).

(e) Suppose the strength of the delta function spikes alters between \( \alpha \) (when \( j \) is even) and \( \alpha/2 \) (when \( j \) is odd), give a qualitative description and a sketch indicating the change that would happen to the particles energy band structure.