DEPARTMENT OF PHYSICS
University at Albany
State University of New York

Comprehensive Field Examination

Part III

Thursday, September 1, 2005
9:00 - 11:00 AM

Instruction:

Answer any three out of four questions

Please check below the three (3) problems you have done and return this sheet with your examination books. Each problem MUST be done in a separate examination book. The problems within each area carry equal weight.

Statistical and Thermal Physics

9. 
10. 
11. 
12. 

Student Identification Code: ____________

NOTE: This same Code is to be used on all sections of the Comprehensive Examination taken in Fall 2005.
Answer three out of four questions

9. Find the change in entropy for each system described below.
   a) A piece of metal with a constant heat capacity $C$ is heated from 250K to 300K.
   b) The same piece of metal is cooled from 250K to 200K.
   c) Two identical boxes are at the same temperature and pressure. Each has volume $V$ and
      each contains $N$ atoms. The atoms are all the same ($A$-atoms). The boxes placed next to
      each other, and a door is opened between the two boxes so the atoms can move from box
      to box.
   d) For case c): After a while, the door between the two boxes is closed.
   e) The same as c), except the atoms in the two boxes are different. ($A$-atoms and $B$-atoms)
   f) For case e): After a while, the door between the two boxes is closed.
Statistical and Thermal Physics

Fall 2005

Answer three out of four questions

10. An impurity site in a solid may be occupied by a single electron with either spin-up or spin-down. The energy levels of the singly-occupied site are

\[ E(\pm) = \varepsilon \pm mB \]

where the \( \pm \) corresponds to the two spin directions, \( B \) is a magnetic field, and \( m \) is the electron's magnetic moment. The site may also be empty. Consider two cases.

I) Electron-electron interactions are not important, so two electrons are allowed to occupy the site at the same time. When two electrons occupy the site, their total energy is the sum of the single-electron energies; \( E(+) + E(-) \)

II) Electron-electron interactions make the energy of the doubly occupied site so large that it is forbidden.

For case I) and case II), assume the probability that the site is empty is \( \frac{1}{2} \).

a) Find the chemical potential \( \mu(I) \) for case I) in terms of \( \varepsilon \) and \( mB \) and \( \beta = 1/(kT) \).
b) Find the chemical potential \( \mu(II) \) for case II) in terms of \( \varepsilon \) and \( mB \) and \( \beta = 1/(kT) \).
c) For which case is the chemical potential larger? That is, what is the sign of \( \mu(I) - \mu(II) \)
Statistical and Thermal Physics

Fall 2005

Answer three out of four questions

11. The average energy of a system in thermal equilibrium is \( \langle E \rangle \). The mean-square deviation from \( \langle E \rangle \) is \( \langle (E - \langle E \rangle)^2 \rangle \).

a) Show that \( \langle (E - \langle E \rangle)^2 \rangle = kT^2 C_v \), where \( C_v \) is the heat capacity at constant volume, and \( T \) is the temperature and \( k \) is Boltzmann’s constant.

b) Use the result from part a) to show that the energy is essentially constant in a macroscopic system in thermal equilibrium.
Answer three out of four questions

12. a) Find the partition function for a single harmonic oscillator with mass \( m \) and angular frequency \( \omega \), treating the oscillator as classical.

b) Find the partition function for a single harmonic oscillator with mass \( m \) and angular frequency \( \omega \), treating the oscillator quantum mechanically.

c) Using the result from part b) find the partition function for a system of \( N \) independent quantum oscillators, all with the same frequency \( \omega \).

d) Using the result from part c), find the internal energy and heat capacity for a system of \( N \) independent harmonic oscillators.
DEPARTMENT OF PHYSICS
University at Albany
State University of New York

Comprehensive Field Examination

Part IV

Thursday, September 1, 2005
1:00 - 3:00 PM

Instruction:

Answer any three out of six questions

Please check below the three (3) problems you have done and return this sheet with your examination books. Each problem MUST be done in a separate examination book. The problems within each area carry equal weight.

Modern Physics

13. __________
14. __________
15. __________
16. __________
17. __________
18. __________

Student Identification Code: __________

NOTE: This same Code is to be used on all sections of the Comprehensive Examination taken in Fall 2005.
Modern Physics

Do three out of six problems

13) Assume the drag force on a car is

\[ F = \frac{1}{2}DA\rho v^2 \]

where

- \( D = \) drag coefficient = 0.5
- \( A = \) cross sectional area of the car = \( 2m^2 \)
- \( \rho = \) mass density of the air
- \( v = \) velocity of the car

Assume the air is an ideal gas of nitrogen molecules, \( N_2 \), and at \( 27^\circ C = 300K \) the density is \( \rho = 1.2kg/m^3 \).

PART I; basics (Using Newtons, Watts, liters, etc.)

a) Find the drag force, \( F \) for the car moving at 30m/sec.
b) Calculate the power needed to maintain this speed, in Watts
c) Gasoline has an energy equivalent of about \( 3.4 \times 10^7 \text{ Joules/liter} \). A gasoline engine is about 18% efficient. Find the number of liters of gasoline needed to drive this car one kilometer.

PART II; American units

a') Use: 1 mile = 1.61km to find the car’s speed (30m/sec) in miles/hour.
b') Use \( 1HP = 746Watts \) to find the horsepower needed to drive this car at 30m/sec.
c') Use one gallon = 3.8 liters to find the number of miles this car would go with one gallon of gas.

PART III; atmospheric variations

d) At 100% relative humidity and \( 27^\circ C = 300K \), about 1% of the atmospheric molecules are water vapor. What will be the change in force needed to maintain the car’s speed if the relative humidity is increased from 0% to 100%. Assume there is no change in the total pressure or the temperature. The atomic masses of nitrogen, oxygen and hydrogen are approximately 7, 8, and 1, respectively.

e) What will be the change in force needed to maintain the car’s speed if the temperature is changed from \( 27^\circ C = 300K \) to \( -23^\circ C = 250K \)? Assume no change in pressure.
Do three out of six problems

14) a) If a 1 TeV proton and 1 TeV anti-proton collide head-on, what is the maximum possible total mass of the products produced by the collision?

b) If a 2 TeV anti-proton collides with a proton contained in a fixed target, what is the maximum possible total mass of the products produced by the collision?

c) Can a free particle decay into a single particle? If yes, give an example; if no, then explain why not.

d) Can a photon undergo pair-production without being in the presence of matter? You must give an explanation to receive credit.

Reminder: The mass of a proton is $\sim 938\, MeV/c^2$. 
Modern Physics

Do three out of six problems

15. Consider Rutherford backscattering from a Ta target.

The Rutherford backscattering differential cross section near $\theta = 180^0$ is

$$\frac{d\sigma}{d\Omega} = \left( \frac{Z_1Z_2e^2}{4E} \right)^2$$

where $Z_1$ and $Z_2$ are the atomic numbers of the projectile and target atoms, and $E$ is the bombarding energy.

In convenient units, $e^2 = 1.44 \times 10^{-13} \text{MeV} \cdot \text{cm}$.

The experimental parameters are as follows:
Scattering angle, $\theta = 180^0$
Detector solid angle, $\Omega = 3 \times 10^{-3} \text{sr}$
Total charge of incident particles, $Q = 10 \mu\text{Coul}$
Beam energy, $E = 2 \text{MeV}$
The electron charge is $e = 1.6 \times 10^{-19} \text{Coulombs}$

The target is a 500Å layer of Ta
The density of Ta is $16.6 \text{grams/cm}^3$.
The atomic number of Ta is $Z = 73$. The atomic mass is $M = 181$
Avagadro’s number is $6.02 \times 10^{23} \text{atoms/gram molecular weight}$
The stopping power of He in Ta is $110eV/\text{[}10^5 \text{atoms/cm}^2\text{]}$

Calculate the following
a) The maximum energy of a backscattered ion.
b) The total number of backscattered ions.
c) The minimum energy of a backscattered ion.
d) If the Ta film is now oxidized to TaO$_2$, describe qualitatively how each of the above [a), b), c)] change.
Do three out of six problems

16. One simplified version of the universe postulates that (on average) objects move away from each other at a speed proportional to the distance between them. That is, for two objects with coordinates $\vec{r}_i$ and velocities $\vec{v}_i$,

$$|\vec{v}_1 - \vec{v}_2| \equiv K |\vec{r}_1 - \vec{r}_2|$$

The "constant" $K$ is roughly the inverse of 15 billion years.

If light is emitted with a frequency $\omega$ by a source moving away from an observer, its observed frequency $\omega'$ will be decreased by the Doppler effect:

$$\omega' = \omega \sqrt{\frac{c-v}{c+v}}$$

Here $v$ is the speed at which the source and observer are moving away from each other, and $c$ is the speed of light.

a) If the light we see from a distant star has its frequency cut in half by the Doppler shift, $\omega' = \frac{1}{2} \omega$.

find the speed at which the star is moving away from us.

b) Given this speed, and the relation $|\vec{v}_1 - \vec{v}_2| \equiv K |\vec{r}_1 - \vec{r}_2|$, find the approximate distance to this star in light years.

In some cases, the separation of distance light sources can be found independently. Suppose we know an object is a distance $x$ from us. It emits light for which

$$\frac{\omega'(x)}{\omega} = \frac{1}{\sqrt{2}}$$

Suppose we know another object is a distance $2x$ from us. It emits light for which

$$\frac{\omega'(2x)}{\omega} = \frac{1}{2}$$

c) Are these observations consistent with;

A) A uniformly expanding universe, as characterized by the equation $|\vec{v}_1 - \vec{v}_2| \equiv K |\vec{r}_1 - \vec{r}_2|$?

B) A universe in which the expansion is accelerating?

C) A universe in which the expansion is decelerating?

Since a guess has a 1/3 probability of being correct, you should present a clear explanation of your choice (A), (B) or (C)) to get credit.

d) Recent observations suggest that our universe is actually characterized by A), B) or C). Which is it?
Modern Physics

Do three out of six problems

17. Nuclear fission continues to be an important popular press topic. Describe nuclear fission. In your description, include answers to the following:

a) What is the source of energy in nuclear fission?

b) The only naturally occurring fissionable isotope is 235U. What makes collecting this material in a form suitable for a reactor or a bomb so difficult?

c) Reactors and bombs can also be made with 239Pu. How is 239Pu produced?

d) Almost all controlled nuclear fission reactors use water as a key component. In addition to a coolant or heat exchange medium, what is the important role of water in a reactor?

e) Describe briefly how breeder reactors work.

f) All common reactor designs make it impossible for fission to proceed uncontrollably, even if the core were to melt. Explain why.

g) Mention at least one of the most important environmental advantages of fission reactors as a source of electrical energy compared to a conventional oil, gas or coal plant.

h) Mention at least one of the most important environmental disadvantages of fission reactors as a source of electrical energy compared to a conventional oil, gas or coal plant.
Do three out of six problems

18. Sketch of a number of effects seen in condensed matter physics are shown in the following pages. They are:

(i) The temperature dependence of the resistivity of a metal.
(ii) The temperature dependence of the heat capacity of an insulator.
(iii) The temperature dependence of the magnetization of a ferromagnet.
(iv) The acoustic phonon frequency (energy) as a function of momentum (wave number).
(v) Electron energy as a function of momentum (wave number).
(vi) The x-ray scattering intensity from a liquid as a function of energy.
(vii) The energy dependence of the Fermi function.

a) Say which diagram (A,B,C,D,E,F,G) goes with which label (i,ii,iii,iv,v,vi,vii). Briefly justify your choices.

b) Give an order of magnitude of the x-coordinate (horizontal) and y-coordinate (vertical) for each graph. Briefly justify your scale choices.
DEPARTMENT OF PHYSICS
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Comprehensive Field Examination

Part V

Saturday, September 3, 2005
9:00 - 11:00 AM

Instruction:

Answer any three out of four questions

Please check below the three (3) problems you have done and return this sheet with your examination books. Each problem MUST be done in a separate examination book. The problems within each area carry equal weight.

Quantum Mechanics

19. ______
20. ______
21. ______
22. ______

Student Identification Code: ______

NOTE: This same Code is to be used on all sections of the Comprehensive Examination taken in Fall 2005.
Quantum Mechanics

Answer three out of four questions

19. The Schrödinger equation for a single electron in the presence of a nucleus with charge $Ze$ is

$$\frac{-\hbar^2}{2m} \nabla^2 \psi(\vec{r}) - \frac{Ze^2}{4\pi\varepsilon_0 r} \psi(\vec{r}) = E \psi(\vec{r})$$

For wave functions which depend only on the distance from the origin, $r$,

$$\nabla^2 \rightarrow \frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr}$$

a) The ground state wave function is of the form

$$\psi(r) = N \exp(-\alpha r)$$

Substitute this wave function into the Schrödinger equation; find $\alpha$ and the ground state energy, $E$.

Using the wave function of part a);

b) Find the normalization constant $N$.

c) What is the angular momentum of the electron?

d) Find the average value of the distance between the electron and the nucleus.

e) Find the most probable position of the electron.
Quantum Mechanics

Answer three out of four questions

20. The states $|1,1\rangle$, $|1,0\rangle$ and $|1,-1\rangle$ represent the spin states of a spin-1 particle having spin angular momentum along the z-axis $S_z = \hbar$, $S_z = 0$ and $S_z = -\hbar$.

Spin-1 particles in a state $|1,1\rangle$ are incident on a Stern-Gerlach device oriented at an angle $\theta$ from the z-axis toward the x-axis. This device transmits only particles with spin $+\hbar$ along the $\theta$ axis (particles in the state $|1,\theta\rangle$). Then the polarizations of these particles are measured along the original z-axis.

a) What fraction of incident particles will be observed with $S_z = \hbar$, $S_z = 0$ and $S_z = -\hbar$?

b) At what angle $\theta$ would you orient the Stern-Gerlach device to maximize the fraction of emerging particles with $S_z = -\hbar$?

NOTE:
A spin-1 particle with spin angular momentum $+\hbar$ along an axis which is at an angle $\theta$ from the z-axis in the x-axis direction is in the state

$$|1,\theta\rangle = \frac{1}{2} (1 + \cos(\theta)) |1,1\rangle + \frac{1}{\sqrt{2}} \sin(\theta) |1,0\rangle + \frac{1}{2} (1 - \cos(\theta)) |1,-1\rangle.$$
Quantum Mechanics

Answer three out of four questions

21. A zero-spin particle of mass $m$ is confined to a box of length $a$, so its wave function vanishes at the ends of the box:

$$\psi(0) = \psi(a) = 0$$

Initially, the potential in the box is zero: $V(x) = 0$.

a) Find the normalized wave functions $\psi_n(x)$ and corresponding energy levels $\varepsilon_n$ for this particle. ($n = 1, 2, 3, \cdots$)

The potential in the box is changed:

$$V(x) \rightarrow V_0 \cos\left(\frac{\pi x}{a}\right)$$

b) Find the ground state and first excited state energy levels $E_1$, $E_2$ to second order in perturbation theory.

c) Find the ground state wave functions $\Psi_1$ to first order in perturbation theory.

d) Assume

$$V_0 = \frac{1}{10} \frac{\hbar^2}{2m} \left(\frac{\pi}{a}\right)^2$$

Give the numerical value of the relative ground state energy shift

$$\frac{E_1 - \varepsilon_1}{V_0}$$

NOTES:
Trigonometric identity:

$$\sin(s) \cos(t) = \frac{1}{2} (\sin(s + t) + \sin(s - t))$$

Perturbation theory:
If $\psi_n(x)$ and $\varepsilon_n$ are the eigenstates and energies of a Hamiltonian $H_0$. Then approximate eigenstates $\Psi_n$ and energies $E_n$ of a perturbed Hamiltonian $H_0 + V(x)$ are

$$\Psi_n = \psi_n + \sum_{m \neq n} \frac{\langle m | V | n \rangle}{\varepsilon_n - \varepsilon_m} \psi_m + \cdots$$

and

$$E_n = \varepsilon_n + \langle n | V | n \rangle + \sum_{m \neq n} \frac{|\langle m | V | n \rangle|^2}{\varepsilon_n - \varepsilon_m} + \cdots$$

where

$$\langle m | V | n \rangle = \int_0^a \psi_m^*(x)V(x)\psi_n(x) \, dx$$

for normalized wave functions.
Quantum Mechanics

Answer three out of four questions

22. A Weber bar was one of the early methods of searching for gravity waves. It is essentially a cooled aluminum cylinder suspended so that it can vibrate in its longitudinal (lengthwise) modes. If a gravity wave hits it, the bar will oscillate very slightly at a resonant frequency, and the distortion can be detected.

Assume the bar has mass 1000kg and its fundamental resonant frequency is 1000Hz. Answer the following questions with algebraic expressions. Then convert your answers to numbers and functions of \( n \) using the following values for Planck’s constant and Boltzmann’s constant.

\[
h = 1.054 \times 10^{-34} \text{ joule} \cdot \text{sec}
\]

\[
k_b = 1.38 \times 10^{-23} \frac{\text{joule}}{K}
\]

a) Treating the bar as a simple harmonic oscillator, what is the vibrational energy in the \( n \)'th quantum state?

b) Treating this vibration mode classically, what is the vibration amplitude \( x_n \) for the bar in its \( n \)'th quantum state?

c) If the system absorbs a graviton (quantized gravity wave) which excites the bar from quantum state \( n \) to quantum state \( n+1 \), what will be the change in the vibration amplitude?

d) If the bar is cooled to 2.7 K, approximately what value would you expect for \( n \) in thermal equilibrium?

e) Why is it reasonable to apply the classical approximation to parts b) and c)?

f) If a detector was designed to see vibration amplitude changes as small as 1/10 atomic radius, could the absorption of a single graviton at 2.7K, as described in c) and d), be observed?
DEPARTMENT OF PHYSICS
University at Albany
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Comprehensive Field Examination

Part VI

Saturday, September 3, 2005
1:00 - 3:00 PM

Instruction:

Answer any two out of three questions

Please check below the two (2) problems you have done and return this sheet with your examination books. Each problem MUST be done in a separate examination book. The problems within each area carry equal weight.

Advanced Quantum Mechanics

23. _________
24. _________
25. _________

Student Identification Code: _________

NOTE: This same Code is to be used on all sections of the Comprehensive Examination taken in Fall 2005.
Answer two out of three questions

23. A particle of mass \( m \) and energy \( E \) moves along the \( x \)-axis and encounters a potential barrier, \( V(x) \), with

\[
V(x) = \begin{cases} 
0 & x < 0 \\
E & 0 \leq x \leq a \\
0 & a < x 
\end{cases}
\]

Note that the barrier height is exactly the same as the particle's energy.

a) Find the probability that the particle will be reflected by this barrier in terms of \( m, E, a \) and other constants.

b) Check to see if the results make sense for \( a \to 0 \) and for \( a \to \infty \).

c) Assume an incident plane wave from the \( -x \) direction, has a reflection probability of \( \frac{1}{2} \) at the barrier. For this case, find (and simplify) the probability density \( P(x) \approx \psi(x)^*\psi(x) \) for all \( x \). Here \( \psi(x) \) is the total wave function. Do not try to normalize \( \psi(x) \).

d) Make a sketch of \( P(x) \) for \( -2a \leq x \leq 2a \).

NOTE:
\[
\sin(2\theta) = 2 \sin(\theta) \cos(\theta)
\]
\[
\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))
\]
Advanced Quantum Mechanics

Answer two out of three questions

24. The linear harmonic oscillator with the Hamiltonian

$$\hat{H}_0 = \hbar \omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right)$$

is subjected to a time-dependent perturbation

$$\hat{H}(t) = K \left( \hat{a}^\dagger + \hat{a} \right)^3 \frac{\exp\left(-i^2 / \tau^2\right)}{\tau \sqrt{\pi}}$$

where $K$ and $\tau$ are constants. The unperturbed eigenequation is

$$\hat{H}_0 |n\rangle = E_n |n\rangle, \quad E_n = \hbar \omega \left( n + \frac{1}{2} \right)$$

where $n = 0, 1, 2, \cdots$ and $|n\rangle$ are the orthonormal eigenstates of the number operator $\hat{N} = \hat{a}^\dagger \hat{a}$ satisfying

$$\langle n | m \rangle = \delta_{n,m}$$

and

$$\hat{a} | n \rangle = \sqrt{n} | n - 1 \rangle, \quad \hat{a}^\dagger | n \rangle = \sqrt{n+1} | n + 1 \rangle$$

When the system is in a state

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} c_n(t) |n\rangle \exp\left(-iE_n t / \hbar\right)$$

the coefficients $c_n(t)$ can be obtained from perturbation theory. If the system is initially $(t \rightarrow -\infty)$ in the state $|m\rangle$, the $c_n(t)$ for $m \neq n$ are given to first order by

$$c_n(t) = \frac{1}{i\hbar} \int_{-\infty}^{t} \langle n | \hat{H}(t') | m \rangle \exp(i(E_n - E_m)t'/\hbar) dt'$$

Assume the system is initially in its ground state, $m = 0$. Find the probability that the system makes a transition to the first excited state as $t \rightarrow \infty$.

HINT: For $\alpha > 0$

$$\int_{-\infty}^{\infty} \exp(-\alpha s^2 + \beta s) ds = \exp\left(\frac{\beta^2}{4\alpha}\right) \sqrt{\frac{\pi}{\alpha}}$$
Answer two out of three questions

25. The quantum numbers $s, l, j, s_z, l_z, j_z$ are used to characterize the state of a particle moving in spherically symmetric potential. The numbers $s, l, j$ correspond to spin angular momentum, orbital angular momentum and total angular momentum. The subscript $z$ characterizes the component of these angular momenta along the $z$-axis. (Other symbols are also used for these quantities.)

a) An electron is characterized by $s_z = 1/2$ and $l = 2$ and $l_z = 2$. What are the allowed values of $j$ and $j_z$ for this electron?

b) An electron with $j = 5/2$, $l = 2$ and $j_z = 1/2$ can be written as a linear combination of states with given values of $s_z$ and $l_z$

$$|j = 5/2; j_z = 1/2; l = 2\rangle = \sum_n c_n |s_z; l_z; l = 2\rangle$$

where $n$ is shorthand notation for all the states which appear in the sum. Identify the states which appear in this sum, and find the coefficients $c_n$.

Note: The lowering operator $J_-$ for states with angular momentum $J$ and $z$-component $J_z$ gives

$$J_- |J, J_z\rangle = \sqrt{J(J + 1) - J_z(J_z - 1)} |J, J_z - 1\rangle$$

For the case of the electron,

$$j_- = l_- + s_-$$