Do as many problems as possible.

1. Suppose $X$ and $Y$ are compact, $T^2$ spaces and $f : X \to Y$ is a bijection (not assumed to be continuous) such that if $C$ is a compact set in $X$ then $f(C)$ is compact in $Y$. Prove that $f$ is a homeomorphism.

2. Let $A_j$ for $j$ in $J$ be a set in $X$. Suppose that $U A_j$ is closed. Prove that $\overline{U A_j} = U \overline{A_j}$.

3. Let $T$ be the usual topology on $R$ and let $V$ be the topology on $R$ where the open set are of the form $(a, \infty)$ for $a$ in $R$ together with the empty set and $R$. Is $R \times R$ with the product topology $T \times Y$ regular?

4. Prove that $X \times Y$ is connected if and only if $X$ and $Y$ are connected.

5. Let $X$ be a compact metric space with metric $d$ and let $A$ be the family of closed sets in $A$. Define a function on $A \times A$ by $e(a, b) = \inf\{r : a \subset N_r(b) \text{ and } b \subset N_r(a)\}$ where $N_r(c)$ is the set of points in $X$ whose distance to $c$ is less than $r$. Prove that $e$ defines a metric on $A$.

6. Let $p : E \to B$ be a covering map where $B$ is connected. Suppose that $p^{-1}(b)$ is a set with $n$-members. Prove that $p^{-1}(b)$ is a set with $n$-members for each $b \in B$.

7. Prove that $\pi_1$ is not abelian.

8. Define a relation $\sim$ on $R^2$ by $(x_1, y_1) \sim (x_2, y_2)$ provided $x_1 + y_1^2 = x_2 + y_2^2$.

   (a) Prove that $\sim$ is an equivalence relation.

   (b) Describe the identification (quotient) space $R^2 / \sim$. 