Do as many problems as you can.

1. Let \( X = \mathbb{R}^2 / \{\text{y-axis}\} \), the plane with the y-axis collapsed to a point, with the quotient topology.
   (a) Is \( X \) Hausdorff? Support your answer.
   (b) Is \( X \) locally compact? Support your answer.

2. Prove that a compact Hausdorff space is normal.

3. Let \( X \) be a compact metric space with metric \( d \) and the property that for all \( t < 1 \), there are pairs of points \( x_t, y_t \) so that \( d(x_t, y_t) = t \). Prove there are points \( x \) and \( y \) so that \( d(x, y) = 1 \).

4. Let \( \sim \) be the equivalence relation on the sphere
   \[
   S^2 = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 = 1\}
   \]
   given by
   \[
   (x, y, z) \sim (-x, -y, z).
   \]
   (In other words, each point is equivalent to the opposite point on the same circle of constant latitude.) Prove that \( S^2 / \sim \) with the quotient topology is homeomorphic to \( S^2 \).

5. (a) Prove that a compact metric space is second countable.
   (b) Let \( G \) be a graph (that is, one-dimensional simplicial complex) having a vertex which is an endpoint of infinitely many edges. Recall that the weak topology on \( G \) is the smallest collection of subsets such that the intersection with each open edge is open within that edge. Show that \( G \) with the weak topology is not metrizable.

6. (a) Give an example of a space which is connected but is not path-connected. You need to explicitly describe the space and prove it is an example.
   (b) Prove that a CW complex is connected if and only if it is path-connected.

7. Let \( p: \tilde{X} \to X \) be a covering map with \( \tilde{X} \) and \( X \) path-connected. Show that the number of points in \( p^{-1}(x) \) is independent of \( x \) in \( X \) and equals the index of the subgroup \( p_* \pi_1(\tilde{X}) \) in \( \pi_1(X) \). This number is called the number of sheets of the covering.

8. (a) Compute the fundamental group of \( \mathbb{R}P^2 \vee S^1 \), the one-point union of the projective plane and a circle.
   (b) Find all 2-sheeted and 3-sheeted coverings of \( \mathbb{R}P^2 \vee S^1 \).
   (c) Find the universal cover of \( \mathbb{R}P^2 \vee S^1 \).