1. State the following theorems:
   Fatou’s lemma,
   Lebesgue dominated convergence theorem,
   Fubini’s theorem,
   Egoroff’s theorem,
   Hölder’s inequality,
   The Radon-Nikodym theorem.

2. State and prove the Monotone convergence theorem (without relying on any of the
   other convergence theorems).

3. Prove that the sum of two measurable functions is a measurable function.

4. Show that if \( f \) is a monotone function on \([a, b]\) then \( f \) has at most a countable number
   of discontinuities.

5. Let \( f_n \) be a sequence of measurable functions such that \( f_n(x) \to f(x) \) almost every-
   where, and suppose that \( \sup \int_0^1 |f_n(x)| dx < \infty \).

   (a) Show that \( f \) is measurable and that \( \int_0^1 |f(x)| dx < \infty \).

   (b) Does \( \lim_{n \to \infty} \int_0^1 f_n(x) dx = \int_0^1 f(x) dx ? \)

6. Show how to construct a non-constant continuous function on \([0,1]\) which is different-
   tiable at each rational point and such that for every rational number \( x, \)
   \( f'(x) = 0 \).

7. Let \( f(x) \) be a measurable function on \([0, \infty)\) such that
   \( \int_0^\infty [f(x)]^n dx = c \) for \( n = 2, 3, 4 \).

   Show that \( f(x) = \chi_A(x) \) almost everywhere for some measurable set \( A \subseteq [0, \infty) \).

8. Show that if \( f \in L^1(X, \mu) \) then
   \( \int_0^\infty \mu\{x : |f(x)| > t\} \ dt = \int_X |f(x)| \ d\mu(x) \).