1. State the following theorems.
   Fatou’s lemma
   Lebesgue Dominated Convergence Theorem
   Lebesgue Monotone Convergence Theorem
   Egoroff’s Theorem
   Minkowski’s Inequality
   The Radon-Nikodym Theorem

2. Prove Fatou’s Lemma from basic principles.

3. Let $E$ be the subset of $[0, 1]$ such that $x \in E$ if and only if there is only one 9 in the decimal expansion of $E$. Prove that $E$ has Lebesgue measure 0.

4. Calculate
   \[ \lim_{h \to \infty} \int_0^1 \frac{h^{3/2}x^{3/2}}{1 + h^2x^2} \, dx \]
   Justify your calculation.

5. Let $\mu$ be a finite measure on the Borel sets of $(-\infty, \infty)$. Let
   \[ f(x) = \int_{-\infty}^\infty e^{itx} d\mu(t) \]
   Prove or give a counterexample: $f(x)$ is uniformly continuous on $(-\infty, \infty)$.

6. Let $f(x) \geq 0$ be a function $[0, 1]$ and let $E = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq f(x)\}$. Prove that if $E$ is a 2-dimensional Lebesgue measurable set than $f$ is a Lebesgue measurable function.

7. Let $f(x)$ be a Lebesgue integrable function such that $\int_0^1 f(x)x^n \, dx = 0$ for all $n \geq 2$.
   Prove or give a counterexample: $f(x) = 0$ almost everywhere.

8. Let $A$ and $B$ be Lebesgue measurable sets of finite non-zero measure. Let
   \[ \varphi(x) = |A \cap (B + x)| \]
   where absolute value denotes Lebesgue measure and $B + x = \{y : y = b + x$ for some $b \in B\}$. Prove or give a counterexample: $\varphi(x)$ is continuous.