1. Suppose that \( f, g : \mathbb{R} \to \mathbb{R} \) and \( \lim_{x \to a} f(x) = b, \lim_{y \to b} g(y) = c. \) Then \( \lim_{x \to a} g(f(x)) = c. \) Prove, or give a counterexample.

2. Let \( f : \mathbb{R} \to \mathbb{R}. \) Prove that the set of irrational numbers can not be the set of discontinuities of \( f. \)

   *Hint.* Prove that the set of discontinuities of \( f \) is a set of type \( F_\sigma \) and the set of irrationals is not.

3. Suppose that \( f \) is a measurable function, and \( g \) is a continuous function and the composition \( f \circ g \) is defined. Is \( f \circ g \) necessarily measurable? Prove, or give a counterexample. What about \( g \circ f \), if it is defined? (again, either prove that it is measurable, or give a counterexample).

4. Let \( \mu \) and \( \nu \) be finite signed measures. Show that there is a signed measure \( \mu \wedge \nu \) which is smaller than \( \mu \) and \( \nu \), but larger that any signed measure which is smaller than \( \mu \) and \( \nu \). Also prove that if \( \mu \) and \( \nu \) are positive, then they are mutually singular if and only if \( \mu \wedge \nu = 0. \)

   *Hint.* Show that \( \mu \wedge \nu = \frac{1}{2}(\mu + \nu - |\mu - \nu|). \)

5. Let \( E \) be a set of finite Lebesgue measure. Show that

\[
\lim_{n \to \infty} m(E \cap [n, +\infty)) = 0.
\]

6. Let \( f \) be a real, integrable function on \([0, 1]\) such that \( \int_0^a f(x) \, dx = 0 \) for all \( a \in [0, 1] \). Show \( f(x) = 0 \) a.e.

7. Let \( f \) and \( g \) be integrable functions on \( \mathbb{R}. \) Define \( f \ast g(x) = \int f(x - y)g(y) \, dy. \) Show \( \int f \ast g(x) \, dx = \int f(x) \, dx \int g(x) \, dx. \) Justify all the steps.

8. (1) Show by direct computation that \( f(x) = x^2 \) is absolutely continuous in \([0, 1]\).

(2) Let \( g(0) = 1 \) and \( g(x) = x \sin(1/x) \) for \( 0 < x \leq 1. \) Is \( g \) of bounded variation? Give a complete justification.

(3) Is every continuous function on \([0, 1]\) absolutely continuous? Justify with reasonable detail.