1. Let \( X_1, X_2, \ldots \), be a sequence of independent, identically distributed random variables with (common) mean \( \mu \) and variance \( \sigma^2 \).

For \( n = 1, 2, \ldots \), let \( Y_n = (1/n)(X_1 + X_2 + \ldots + X_n) \).

For \( n = 1, 2, \ldots \), let \( W_n = (Y_n - \mu)/\sigma \).

For \( n = 1, 2, \ldots \), let \( Z_n = (\sqrt{n})W_n \).

For each of the sequences \( \{X_n\} \), \( \{Y_n\} \), \( \{W_n\} \) and \( \{Z_n\} \) state whether or not it converges in probability and/or in distribution and, if so, to what. Give brief justifications for your answers.

2. Let \( Z \) be a standard 0, 1 normal and let \( W \) be a chi square with \( n \) degrees of freedom.

a. Write down the density functions of \( Z \) and \( W \).

b. If \( Z \) and \( W \) are independent, find the density function of

\( (\sqrt{n})Z/\sqrt{W} \). What name is given to this distribution?

3. Let \( X \) have a gamma distribution with parameters \( \alpha = 3 \) and \( \beta = \theta > 0 \).

a. Find the Fisher information \( I(\theta) \).

b. If \( X_1, X_2, \ldots , X_n \) is a random sample from this distribution, find the maximum likelihood estimator of \( \theta \) and find the efficiency of this maximum likelihood estimator.
4. Let $X_1, X_2, \ldots, X_n$ be independent uniform distributions on the real interval $[0, \theta]$, where $\theta$ is unknown.

   a. Find a sufficient statistic for $\theta$ and justify your answer.

   b. Is your sufficient statistic an unbiased estimator of $\theta$? Prove your answer.

5. Suppose $X_1, X_2, \ldots, X_n$ are an independent random sample from a Bernoulli distribution with success parameter $p$, where $p$ itself has the prior distribution $f(p) = 12p^2(1-p)$, where $0 < p < 1$.

   a. Find the mean of the prior distribution of $p$.

   b. Find the posterior distribution of $p$.

   c. If $n = 6$ and the $X$'s are 0, 1, 1, 1, 0, 0, find the mean of the posterior distribution of $p$. 