University at Albany

Department of Mathematics and Statistics

Preliminary Examination

Mathematical Statistics

June, 2006

Do as many problems as possible!

1. Let $X_1, \ldots, X_n$ denote a random sample from a normal distribution with mean $\mu$ and variance 9. Find the value of $n$ such that the 95 percent confidence interval for $\mu$ will be $(\bar{x} - 0.196, \bar{x} + 0.196)$. Also find the value of $n$ so that the 95 percent confidence interval for $\mu$ will be $(\bar{x} - 0.098, \bar{x} + 0.098)$.

2. Suppose

$$f_1(x) = \begin{cases} c_1(1 - x) & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$f_2(x) = \begin{cases} c_2(1 + x) & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

are continuous probability density functions where $c_1$ and $c_2$ are constants.

a. Find the constants $c_1$ and $c_2$.

b. Let $X_1, \ldots, X_{100}$ denote a random sample from a distribution with probability density function $f(x)$. Describe a best test of the hypothesis $f(x) = f_1(x)$ against the hypothesis $f(x) = f_2(x)$.

3. Suppose $W$ is a standard normal random variable and $V$ is a chi-square random variable with 10 degrees of freedom. Suppose also that $W$ and $V$ are independent; thus the joint probability density function of $W$ and $V$ is

$$h(w, v) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-w^2/2} & \text{if } v > 0 \\ \frac{1}{\Gamma(5)^2} \frac{1}{v^4} e^{-v/2} & \text{if } v > 0 \\ 0 & \text{otherwise} \end{cases}$$

Let $T = W/\sqrt{V/10}$ and $U = V$. Find the joint probability density function of $T$ and $U$, and use this result to find the probability density function of $T$. 
4. In a sample of size 10 from a normal distribution with unknown mean $\mu$ and unknown variance, the observed values have sample mean 2.38 and sample variance 4.23. Describe how to find a 99 percent confidence interval for $\mu$; this description involves a value $b$ from a commonly available table. You need not give its value, but you should describe which table you would use and how you would find the value within the table.

5. Let $X_1, \ldots, X_n$ be a random sample from the normal distribution $N(0, \theta)$ where $0 < \theta < \infty$. Is $\sum_{i=1}^n X_i^2$ a sufficient statistic for $\theta$? Justify.

6. Let $X_1, \ldots, X_{2006}$ be i.i.d. random variables which are uniform on the interval $(0, 2)$. Let $Y_1, \ldots, Y_{2006}$ be the order statistics of this random sample. Find, with justification, the distribution function for $Y_{2006}$, and use this function to find the probability density function of $Y_{2006}$. Then find the expected value of $Y_{2006}$.

7. Suppose $X_1, \ldots, X_n$ represents a random sample from a distribution with probability density function

$$f(x; \theta) = \begin{cases} 1/\theta & \text{if } 0 < x \leq \theta \text{ and } 0 < \theta < \infty \\ 0 & \text{otherwise.} \end{cases}$$

Find the maximum likelihood estimator of $\theta$. Is it unbiased? Is it consistent? Justify!