1. Let $X_1, \ldots, X_n$ denote a random sample from a normal distribution with mean $\mu$ and variance 16. Find the value of $n$ so that the 95 percent confidence interval for $\mu$ will be $(\bar{x} - 0.98, \bar{x} + 0.98)$.

2.a. Suppose $X$ is a Poisson random variable with mean $\theta$. Find the moment generating function for $X$.
2.b. Suppose $X$ and $Y$ are independent Poisson random variables with means $\theta_X$ and $\theta_Y$, respectively. What kind of random variable is $X + Y$? Include any relevant parameters, and justify your response.

3. Suppose $X_1, \ldots, X_9$ form a random sample from a distribution which is uniform on $(-1,1)$. Let $Y_1, \ldots, Y_9$ be the order statistics of this random sample. Find the distribution function of $Y_5$, and use this to find the probability density function of $Y_5$.

4. There is an experiment with 3 distinct outcomes $A$, $B$, and $C$. You wish to test the hypothesis $H_0 : P(A) = 0.6, P(B) = 0.3$, and $P(C) = 0.1$ against all other hypotheses. You perform this experiment 500 times. In doing so, you find that $A$ occurs 320 times, $B$ occurs 116 times, and $C$ occurs the remaining times. Carefully describe a test which determines whether you may reject $H_0$ at the approximate 1 percent significance level. This test involves a value from a commonly available table; since you don’t have the table, describe where to find this value and how you would use the value if you had it.
5. Let $X_1, \ldots, X_{100}$ denote a random sample from a distribution with probability density function

$$f(x; \theta) = \begin{cases} 
1/\theta & \text{if } 0 < x < \theta \\
0 & \text{otherwise}
\end{cases}$$

where $\theta > 0$ is a parameter.

a. Let $Y = \max(X_1, \ldots, X_{100})$. Is $Y$ a sufficient statistic for $\theta$? Justify your answer.

b. What is the maximum likelihood estimator for $\theta$? Is this estimator unbiased? Justify your answer.

6. Suppose

$$f_1(x) = \begin{cases} 
c_1x & \text{if } 0 < x < 1 \\
0 & \text{otherwise}
\end{cases}$$

and

$$f_2(x) = \begin{cases} 
c_2x^2 & \text{if } 0 < x < 1 \\
0 & \text{otherwise}
\end{cases}$$

are continuous probability density functions where $c_1$ and $c_2$ are constants.

a. Find the constants $c_1$ and $c_2$.

b. Let $X_1, \ldots, X_{400}$ denote a random sample from a distribution with probability density function $f(x)$. Describe a best test of the hypothesis $f(x) = f_1(x)$ against the hypothesis $f(x) = f_2(x)$.

7. Let $X_1$ and $X_2$ be independent standard normal random variables. Suppose $Y_1 = X_1/X_2$ and $Y_2 = X_2$.

a. Find the joint probability density function of $Y_1$ and $Y_2$.

b. Use part a to find the probability density function of $Y_1$. 