1. Suppose \( X_1, \ldots, X_{200} \) form a random sample from a distribution which is uniform on \((0, 1)\). Let \( Y_1, \ldots, Y_{200} \) be the order statistics of this random sample. Find the distribution function of \( Y_1 \), and use this to find the probability density function of \( Y_1 \). Then find the expected value of \( Y_1 \).

2. Let \( X_1, \ldots, X_n \) represent a random sample from a distribution with probability density function
\[
f(x; \theta) = \begin{cases} 
\theta x^{\theta-1} & \text{if } 0 < x < 1 \\
0 & \text{otherwise}
\end{cases}
\]
where \( \theta > 0 \) is a parameter. Find the maximum likelihood estimator of \( \theta \).

3. There is an experiment with 4 distinct outcomes \( A, B, C, \) and \( D \). You wish to test the hypothesis \( H_0 : P(A) = 0.50, P(B) = 0.30, P(C) = 0.15, P(D) = 0.05 \) against all other hypotheses. You perform this experiment 1000 times. In doing so, you find that \( A \) occurs 453 times, \( B \) occurs 320 times, \( C \) occurs 142 times, and \( D \) occurs the remaining times. Carefully describe a test which determines whether you may reject \( H_0 \) at the approximate 1 percent significance level. This test needs a value from a commonly available table; since you don’t have the table, describe where to find this value and how you would use it.

4. Let \( X \) be a standard normal distribution. What kind of distribution does \( X^2 \) have? Prove your answer.
5. a. Is the sum of the observations of a random sample of size \( n \) from a Poisson distribution with parameter \( \theta > 0 \) a sufficient statistic for \( \theta \)? Justify.

b. Consider the maximum of the observations of a random sample of size \( n > 1 \) from a distribution with probability distribution function

\[
f(x; \theta) = \begin{cases} 
  e^{-(x-\theta)} & \text{if } x > \theta \\
  0 & \text{otherwise}
\end{cases}
\]

with real parameter \( \theta \). Is this maximum a sufficient statistic for \( \theta \)? Justify.

6. You have a random sample of size \( n \) from a normal distribution with unknown mean \( \theta \) and variance 100. You want to find \( n \) large enough so that the length of the confidence interval (from left endpoint to right endpoint) is at most 0.196. Find such a value of \( n \) so that \( n \) is as small as possible. If you instead were willing to have a confidence interval with twice this length, what would you need to do to \( n \)? (Note: All confidence intervals in this problem are 95 percent confidence intervals.)