1. Find an analytic function $f(z)$ whose real part is $(z = x + iy)$.
\[ \text{Re} \, f(z) = xy - 10. \]
Does such a function exist? Justify your answer.

2. Find the general form of an entire function $f(z)$ satisfying
\[ |f(z)| \leq A + B|z|^{3/2}, \] where $A$ and $B$ are constants.

3. Find the general form of a function $f(z)$ which is analytic inside the ellipse $D$ ($z = x + iy$)
\[ \frac{x^2}{16} + \frac{y^2}{9} = 1, \]
continuous in $\partial D$, and
\[ \text{Im} \, f(z) = -5 \quad (z \in \partial D) \]

4. Find a conformal mapping from $\mathbb{C}\setminus\{[0, +\infty)\}$ to the unit disk.

5. 1. Prove that for any polynomial $p$ and any $a \in \Delta$
\[ p(a) = \frac{1}{2\pi} \int_0^{2\pi} \frac{p(e^{i\theta})}{1 - e^{-i\theta}a} d\theta \]
2. Deduce from 5.1 that
\[ |p(a)| \leq \left[ \frac{1}{(1 - |a|^2)^2} \int_0^{2\pi} |p(e^{i\theta})|^2 d\theta \right]^{1/2} \]

6. Let $f$ be analytic in the unit disk and map the unit disk into itself given $f(1/2) = 0$. Prove that $|f'(1/2)| \leq \frac{4}{3}$. 
7. Let 
\[ f(z) = \frac{1}{z} \cdot \frac{1 - 2z}{z - 2} \cdot \ldots \cdot \frac{1 - 10z}{z - 10} \]

Find \( \int_{|z|=100} f(z) \, dz \).

8. Let \( f(z) \not\equiv 0 \) be a meromorphic function in \( \mathbb{C} \) such that
\[ |f(z)| = 1 \quad (|z| = 1) \]

and
\[ f \left( \frac{1}{2} \right) = 0. \]

Can \( f \) be an entire function?