1. Let \( f \) be analytic in a nonempty connected open set \( U \). Let \( F \) be a nonconstant entire function. Show that if \( F(f(z)) = 0 \) for all \( z \) in a neighborhood of some \( z_0 \in U \), then \( f \) is constant in \( U \).

2. (a) Find all constants \( c_1 \) and \( c_2 \) so that the functions

\[
f_1(z) = c_1 z \quad \text{and} \quad f_2(z) = \frac{c_2}{z}
\]

define conformal self-maps of the annulus \( \mathcal{A} = \{ z \in \mathbb{C} : a < |z| < b \} \) (0 < \( a < b \) are given constants).

(b) Prove that there are no other conformal self-maps of \( \mathcal{A} \).

3. Evaluate

\[
\int_{\gamma} \frac{1 - \cos z}{(e^z - 1) \sin z} \, dz
\]

where the path \( \gamma \) is the circle \( |z| = e \) traversed once counterclockwise.

4. For \( n \in \mathbb{N} \) show that

\[
\int_{\Delta} \left| \frac{1 - z^n}{1 - z} \right|^2 \, dxdy = \pi \left( 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \right)
\]

5. Let \( f \) be analytic in \( \Delta \), and let \( f(\Delta) \subseteq \Delta \). Prove that if \( f(0) = 0 \) and \( f(a) = a \) for some \( a \neq 0 \), then \( f(z) = z \).

6. Let \( f \) be analytic in \( \Delta \). Show that

\[
\sup_{z \in \Delta} (1 - |z|^2) \, |f'(z)| \leq \sup_{z \in \Delta} |f(z)|.
\]
7. Let \( f(z) \) be analytic in \( \Delta \). Suppose

\[
\lim_{r \to 1} \int_0^{2\pi} |f(re^{i\theta})| \, d\theta = 0.
\]

Show that \( f \equiv 0 \).

8. Prove that the zero set \( S \) of \( e^z + z \):

\[
S = \{ z \in \mathbb{C} : e^z + z = 0 \}
\]

is nonempty: \( S \neq \emptyset \).

**Bonus.** Prove that \( S \) is an infinite set.

9. Find \( w = f(z) \) that maps \( \Delta \) conformally onto the strip \( |\text{Im} \, w| < \frac{\pi}{2} \) so that \( f(0) = 0 \) and \( f'(0) > 0 \).