Let \( C \) be the set of complex numbers and let \( D = \{ z \in C : |z| < 1 \} \).

1. Let \( U : D \to D \) be harmonic and \( f : D \to D \) be analytic. Prove or disprove the following:
   (1) \( f \circ U \) is harmonic.
   (2) \( U \circ f \) is harmonic.

2. Show that there exists an unbounded analytic function \( f \) on \( D \) such that
   \[
   \int_D |f'(z)|^2 \, dA(z) < +\infty,
   \]
   where \( dA \) is area measure on \( D \).

3. Suppose \( f \) is analytic in \( D - \{0\} \) and unbounded near \( z = 0 \). If the function \( |z|^{\sqrt{2}} f(z) \) is bounded at \( z = 0 \), show that
   \[
   \lim_{z \to 0} |z|^{\sqrt{2}} f(z) = 0 \quad \text{and} \quad \lim_{z \to 0} |z|^{\sqrt{2}/2} f(z) = \infty.
   \]

4. Let \( X \) be the space of analytic functions \( f \) in \( D \) such that
   \[
   \|f\| = \sup_{z \in D} (1 - |z|^2)|f(z)| < +\infty.
   \]
   If \( \{f_n\} \) is a sequence of functions in \( X \) such that \( \|f_n - f_m\| \to 0 \) as \( n, m \to +\infty \), show that there exists a function \( f \in X \) such that \( \|f_n - f\| \to 0 \) as \( n \to +\infty \).

5. Let \( f \) be analytic in \( D \). Show that
   \[
   \sup_{z \in D} (1 - |z|^2)|f'(z)| \leq \sup_{z \in D} |f(z)|.
   \]

6. Suppose \( f \) is analytic in \( D \). For \( z \in D \) and \( 0 < r < 1 - |z| \) let \( B(z, r) = \{ w \in D : |z - w| < r \} \). Show that
   \[
   |f(z)|^\pi \leq \frac{1}{\pi r^2} \int_{B(z, r)} |f(w)|^\pi \, dA(w),
   \]
   where \( dA \) is area measure on \( D \).

7. Evaluate the integral
   \[
   I = \int_{|z|=\pi} \frac{\sin z}{z \cos z} \, dz.
   \]

8. Suppose \( \{a_n\} \) is a sequence in \( D - \{0\} \) with \( \sum (1 - |a_n|) < +\infty \). Show that
   \[
   \prod_{n=1}^{\infty} \frac{|a_n|}{a_n} \frac{a_n - z}{1 - \overline{a_n}z}
   \]
   converges (uniformly on compact sets) to an analytic function in \( D \).