1. Do both of the following:
   (a) Sketch the set of points $z$ satisfying $|z - i|^2 + |z + i|^2 < 2$.
   (b) Evaluate $(1 + i)^{2i}$.

2. Let $f(z) = z + \frac{z^2}{2}$. Find the area of $f(\mathbb{D})$.

3. Construct a nonlinear fractional linear map $\phi(z) = \frac{az + b}{cz + d}$, $c \neq 0$, such that $\phi(\phi(\phi(z))) = z$.

4. State and prove Rouche's theorem.

5. Determine the number of solutions to the equation $z - 2 - e^{-z} = 0$ in the right half plane $\mathbb{H} = \{z \in \mathbb{C} : \text{Re}(z) > 0\}$.

6. Let $f$ be a nonconstant entire function with $f(z) \neq 0$ everywhere. Show that the set $\{z \in \mathbb{C} : |f(z)| < 1\}$ is unbounded.

7. Suppose that $f(z)$ is analytic on the closed unit disk $\overline{\mathbb{D}}$ and $1 < |f(z)| < M$ for $|z| = 1$, while $f(0) = 1$. Show that $f(z)$ has a zero in $\mathbb{D}$, and that such zero $z_0$ satisfies $|z_0| > 1/M$. (Hint: for the second assertion, consider $\psi(f(z)/M)$, where $\psi$ is a fractional linear transform mapping $1/M$ to 0 and $\mathbb{D}$ to $\mathbb{D}$, then use Schwarz's lemma).

8. Do one of the following two problems.
   (a) Show that $\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2} \, dx = \pi$.
   (b) Evaluate $\int_0^\infty \frac{\log x}{x^a (x+1)} \, dx$, $0 < a < 1$. 