1. Show that $u = e^x(x \cos y - y \sin y)$ is harmonic in the complex plane in 2 ways:
   A. From the definition of harmonic.
   B. By exhibiting an entire function $f$ such that $u = Re f$.

2. A. State Schwarz’s Lemma.
   B. State the Riemann Mapping Theorem.
   C. Prove uniqueness in the Riemann Mapping Theorem.

3. Let $a$ and $b$ be real numbers with $a > b > 1$.
   A. Show that $b^z$ can be defined as an entire function such that $b^0 = 1$.
   C. Let $n$ be a positive integer. Show that the equation $b^z = a z^n$ has $n$ solutions in $|z| < 1$.

4. Let $C_\mathcal{E} = \{ \mathcal{E} e^{i\theta} : 0 \leq \theta \leq \pi \}$ denote the semicircle traversed clockwise.
   A. Calculate $\int_{C_\mathcal{E}} \frac{1}{z} \, dz$.
   B. Determine $\lim_{\mathcal{E} \to 0} \int_{C_\mathcal{E}} \frac{1}{z(z^2 + 1)} \, dz$.
   C. Show that $\lim_{\mathcal{E} \to 0} \int_{C_\mathcal{E}} \frac{e^{iz}}{z(z^2 + 1)} \, dz = -\pi i \left[ \text{Consider } e^{iz} - 1. \right]$

5. Map the region bounded by the circles $|z| = 1$ and $|z + 1| = 2$ conformally onto the open unit disk.

6. A. Determine the region of convergence of the series
   
   $$1 + \frac{2z}{1+z} + \frac{3z^2}{(1+z)^2} + \ldots + \frac{(n+1)z^n}{(1+z)^n} + \ldots$$
   
   B. By summing the series show that the series actually represents a polynomial in its region of a convergence.
7. Let Ω be a bounded domain. Let \( F \) be the family of functions which are analytic in Ω and map Ω into itself.

   A. Show that \( F \) has locally bounded derivatives.

   B. Is \( F \) closed in the topology of uniform convergence on compact subsets of Ω?

        Justify your answer.

8. Let \( I = \int_{-\infty}^{\infty} \frac{\sin x}{x(x^2 + 1)} \, dx \).

   A. Explain why \( I \) is absolutely convergent.

   B. Incorporate the semicircle and the result from Problem 4 in a contour integration argument to evaluate \( I \).