1. Let $s = (12)(34)(56789)$ be an element of the symmetric group $S_9$. Describe the centralizer $C_{S_9}(s)$ and compute its order.

2. Show that there is no simple group of order 24.

3. Let $K = \mathbb{Q}(i, \sqrt{2}, \sqrt{3})$. Compute a basis for $K$ as a vector space over $\mathbb{Q}$ and describe the Galois group of $K$ over $\mathbb{Q}$.

4. Let $L$ be the splitting field of $x^4 - 7$ over $\mathbb{Q}$, the rationals. What is its Galois group? Describe all fields intermediate between $L$ and $\mathbb{Q}$, and determine which of them are normal over $\mathbb{Q}$.

5. Let $f(T) \in \mathbb{Q}[T]$ be an irreducible polynomial of prime degree $p \geq 3$ with all but two roots in $\mathbb{R}$ (the reals). Show that the Galois group of $f(T)$ over $\mathbb{Q}$ is isomorphic to $S_p$.

6. Let $G$ be a finite subgroup of the multiplicative group $F^*$ of nonzero elements in a field $F$. Show that $G$ is cyclic.

7. Let $f: U \to V$ and $g: V \to U$ be homomorphisms of vector spaces over a field $K$ such that $fg = \text{id}_V$. Show that $U = \text{Ker}(f) \oplus \text{Im}(g)$.

8. Let $A$ be a matrix with entries in $\mathbb{Q}$ whose characteristic polynomial is $(x+1)^3(x+2)^4$ and whose minimal polynomial is $(x+1)^2(x+2)^2$. Find
   (a) all possible sequences of (polynomial) invariant factors of $A$;
   (b) all possible rational canonical forms of $A$;
   (c) all possible Jordan normal forms of $A$.

9. Given is a fixed prime $p \geq 2$. Construct a non-abelian group of order $p^3$. 
