Let $a_1, a_2, \ldots, a_n, \ldots$, be a sequence of real numbers, which form an arithmetic progression with common difference $d$. For every $n \in \mathbb{N}$, let $S_n = a_1 + a_2 + \cdots + a_n$. Further, for $x \in \mathbb{R}$ let $f(x) = x^2 + (2p - 1)x + 1$, where $p \in \mathbb{R}$, and assume that

$$2S_n + 1 = f(n) \quad (1)$$

for all $n \in \mathbb{N}$. Suppose also that the roots of the equation $f(x) = 0$ are real. If, lastly,

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 - a_1a_2a_3a_4 = \frac{15}{16}, \quad (2)$$

find $a_1, d$, and $p$. 