Let $x$, $y$, and $m$ be real numbers such that
\[
\begin{align*}
| & x + y = 2m - 1 \\
| & x^2 + y^2 = m^2 + 2m - 3.
\end{align*}
\]
For what value(s) of $m$ does $x \cdot y$ achieve its maximum and minimum values, if they exist?

**Solution:**
We have that
\[
x \cdot y = \frac{(x + y)^2 - (x^2 + y^2)}{2} = \frac{(2m - 1)^2 - (m^2 + 2m - 3)}{2} = \frac{3m^2 - 6m + 4}{2}.
\]
Now, $\frac{3m^2 - 6m + 4}{2}$ is a quadratic polynomial in $m$ with a positive coefficient in front of $m^2$. Therefore it achieves its minimum at $m_0 = 1$ and has no maximum value.

Thus the minimum value of $x \cdot y$ is $\frac{3m_0^2 - 6m_0 + 4}{2} = \frac{3 \cdot (1)^2 - 6 \cdot (1) + 4}{2} = \frac{1}{2}$ and $x \cdot y$ has no maximum value.