Solve the equation
\[ \frac{1}{2n + nx} - \frac{1}{2x - x^2} = \frac{2(n + 3)}{x^3 - 4x}, \]
where \( n \) is a real parameter.

**Solution:**

The domain of the parameter is \( D_n = \{n : n \neq 0\} \) and the domain of the variable is \( D_x = \{x : x \neq 0, \pm 2\} \).

\[
\frac{1}{2n + nx} - \frac{1}{2x - x^2} = \frac{2(n + 3)}{x^3 - 4x}
\]
\[
\frac{1}{(2 + x) n} + \frac{1}{(x - 2) x} - \frac{2(n + 3)}{x (x - 2) (x + 2)} = 0
\]
\[
x (x - 2) + (2 + x) n - 2n (n + 3) = 0
\]
\[
x^2 + (n - 2) x - 2n^2 - 4n = 0
\]

The discriminant of this equation is
\[
D = (n - 2)^2 - 4 (-2n^2 - 4n) = n^2 - 4n + 4 + 8n^2 + 16n = 9n^2 + 12n + 4 = (3n + 2)^2.
\]

Therefore the roots of the equation are
\[
x_{1,2} = \frac{2 - n \pm \sqrt{(3n + 2)^2}}{2} = \frac{2 - n \pm (3n + 2)}{2} = \begin{cases} \frac{n + 2}{2} & \text{if } n + 2 \neq -2, \ 2n. \end{cases}
\]

Now, \( n + 2 \in D_x \) if and only if \( n + 2 \neq 0, n + 2 \neq 2, \) and \( n + 2 \neq -2, \) which is equivalent to \( n \neq -4, -2, 0. \)

Similarly, \( -2n \in D_x \) if and only if \( -2n \neq 0, -2n \neq 2, \) and \( -2n \neq -2, \) which is equivalent to \( n \neq -1, 0, 1. \)

Thus we have that
• if $n = 0$, the equation is not defined
• if $n = -4$ or $n = -2$ the solution to the equation is $x = -2n$
• if $n = -1$ or $n = 1$, the solution to the equation is $x = n + 2$
• if $n \neq -4, -2, -1, 1$, then the solutions to the equation are $x_1 = n + 2$ and $x_2 = -2n$. 

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