What is the relationship between the natural numbers $n$ and $k$ if the identity

$$1 + x + x^2 + \cdots + x^n = (1 + x)(1 + x^2)(1 + x^4)\cdots\left(1 + x^{2^k}\right)$$

holds for every $x \in \mathbb{R}\setminus\{\pm1, 0\}$?