High School Math Problems
2016
Week 13
Problem and Solution

Prove that the remainder in division by 8 of the square of any odd number is 1.

Solution:

Let $a = 2k + 1$, where $k \in \mathbb{Z}$.

Then

$$a^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 4k(k+1) + 1.$$

Now, k and $k+1$ are consecutive integers and therefore one of them is even.

Therefore $k(k+1) = 2l$ for some $l \in \mathbb{Z}$.

Therefore

$$a^2 = 8l + 1,$$

from which we clear see that the remainder in division by 8 of a^2 is 1.