Prove that the remainder in division by 8 of the square of any odd number is 1.

Solution:
Let \( a = 2k + 1 \), where \( k \in \mathbb{Z} \).
Then
\[
a^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4k(k + 1) + 1.
\]
Now, \( k \) and \( k + 1 \) are consecutive integers and therefore one of them is even.
Therefore \( k(k + 1) = 2l \) for some \( l \in \mathbb{Z} \).
Therefore
\[
a^2 = 8l + 1,
\]
from which we clearly see that the remainder in division by 8 of \( a^2 \) is 1.