Prove that the altitudes of a triangle intersect in one point.

Proof. Let $ABC$ be an arbitrary triangle. Let $D$ be the point of intersection of the altitudes through the vertices $A$ and $C$.

Then, from the problem from Week 5, we have that

$$\overrightarrow{AB} \cdot \overrightarrow{CD} + \overrightarrow{BC} \cdot \overrightarrow{AD} + \overrightarrow{CA} \cdot \overrightarrow{BD} = 0.$$ 

But

$$\overrightarrow{AB} \cdot \overrightarrow{CD} = 0$$ since $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are orthogonal

and

$$\overrightarrow{BC} \cdot \overrightarrow{AD} = 0$$ since $\overrightarrow{BC}$ and $\overrightarrow{AD}$ are orthogonal.

Therefore

$$\overrightarrow{CA} \cdot \overrightarrow{BD} = 0$$

and thus the altitude of the triangle $ABC$ through the vertex $B$ passes through $D$. □