# High School Math Problems
# 2015
# Week 30
# Problem and Solution

Solve the equation

$$\log_{12} \log_2 \log_3 \frac{3}{2-4x} = 0.$$

**Solution:**
For $x$ to be in the domain of the equation, we must have that

$$\left| \begin{array}{l} x \neq \frac{1}{2} \\ \frac{3}{2-4x} > 0 \\ \log_3 \frac{3}{2-4x} > 0 \\ \log_2 \log_3 \frac{3}{2-4x} > 0. \end{array} \right.$$

From this obtain the system

$$\left| \begin{array}{l} x \neq \frac{1}{2} \\ x < \frac{1}{2} \\ \frac{3}{2-4x} > 1 \\ \log_3 \frac{3}{2-4x} > 1 \end{array} \right.$$

and therefore further the system

$$\left|\begin{array}{l} x \neq \dfrac{1}{2} \\ x < \dfrac{1}{2} \\ \dfrac{3}{2-4x} > 1 \\ \dfrac{3}{2-4x} > 3. \end{array}\right.$$

Solving for $x$, we therefore obtain that the domain $D_x$ of the equation is given by

$$D_x = \left\{ x : \frac{1}{4} < x < \frac{1}{2} \right\}.$$

Since

$$\log_{12}\left(\log_2 \log_3 \frac{3}{2-4x}\right) = 0$$

it follows that

$$\log_2 \log_3 \frac{3}{2-4x} = 1.$$

From

$$\log_2\left(\log_3 \frac{3}{2-4x}\right) = 1$$

we have that

$$\log_3 \frac{3}{2-4x} = 2.$$

From the last equation we now obtain that

$$\frac{3}{2-4x} = 9$$

and solving for $x$ we therefore have that

$$x = \frac{5}{12}.$$

Since $\dfrac{5}{12} \in D_x$, it follows that $x = \dfrac{5}{12}$ is the solution to the equation.