In this paper, I adapt the game-theoretical model of Zuckerman (1985) to include the decision to take on a part-time job while receiving unemployment benefits. The optimal stopping rule is defined, as is the rule for accepting part-time work while receiving unemployment compensation. It is shown that an increase in the value of part-time work actually decreases its appeal, due to the effects on reservation wages.

I. Introduction

The rationale for unemployment insurance is rather straightforward. One, it seeks to smooth consumption for workers during times of economic stress. This, in turn, encourages the worker to make a good job choice. Since its inception in 1935, unemployment insurance has been the focus of scrutiny, with special interest in the effects on social welfare and employment. Yoder (1931) preempted the implementation of UI law with a discussion of the increased unemployment and reduced wages that would accompany compulsory insurance. Rorty (1936) put it even more bluntly: “The unfortunate situation, today, is that the average worker has been promised the rainbow with a pot of gold at the end.” More recently, authors have sought to measure the incentive effects of unemployment insurance and the return to full-time work.

One aspect of the unemployment insurance system that has not been analyzed in detail has been the issue of part-time work. When an unemployed worker is receiving unemployment benefits and searching for a job, he is allowed to take on a part-time job in the meantime and still receive unemployment benefits. The idea of this wage
disregard is to encourage work. In the body of unemployment literature, there are only a handful of notable examples of research on part-time work. Munts (1970) measured the disincentive effects of a certain kind of disregard in Wisconsin, Michigan, and Nebraska and suggested that in the face of extreme disincentives, workers will choose to cut back on working part-time. McCall (1996) studied the effects of varying the size of the disregard and Kyyra (2008) used Finnish data to estimate the likelihood of returning to a full-time job given that the worker took on a part-time job during his search, and found that partial unemployment associated with short-term jobs taken on during an unemployment spell facilitate the transition back to full-time work. Gerfin and Lechner (2002) find results similar to those of Kyyra’s in a Switzerland study.

One curious aspect of the American unemployment insurance system that is addressed briefly in Munts but not McCall is the type of part-time wage disregard that the government employs. Currently, in 39 states, this disregard is some fixed dollar amount, and above that, additional hours worked at the part-time job will lead to a dollar-for-dollar reduction in benefits. The remaining states and Washington, DC employ a partial earnings disregard. In a partial retained earnings system, the unemployed worker’s unemployment compensation is reduced by a fraction of a dollar for every dollar he earns at a part-time job. These two schemes are illustrated in Figure 1. The fact that states have been slowly adopting this partial disregard over time leads to the obvious question of whether or not this system is any better at either encouraging return to work or decreasing the costs of the system. This paper seeks to close that gap, borrowing from the reservation wage model developed by Zuckerman (1985).
The main issue comparing the effects of either system is that part-time work detracts from searching for a full-time job and extends the benefit period. So encouraging working part-time has the possible effect of delaying return to full-time work. On the other hand, it is possible that increased part-time work has positive benefits, such as a stronger commitment to the workforce and increased income. The effect of allowing more job offers. Since the full retained earnings benefit scheme is flat beyond the disregard, which represents no additional income for an additional hour of work, there is the possibility that a worker chooses a sub-optimal part-time work schedule, which may then increase the cost of the program, as suggested by Munts (1970).

In this paper, we will compare changes in the unemployment benefit through direct changes in the benefit, and through changes in the replacement rate \( r \). It is found that an increase in the unemployment benefit, either through the increase in the initial payment or the part-time wage replacement rate, decreases the appeal of part-time work, most likely because of the effects on reservation wages.

The paper is organized as follows. Part II introduces the model, most of which is adapted from Zuckerman. Part III presents a general solution. Part IV presents a numerical example. Part V presents some concluding comments.

II. The Model

In a given period, an individual is endowed with 1 unit of time that is inelastically devoted to employment activities. When he is full-time employed, he works the entire time in that period, the period denoted by \( n \), and when he is unemployed, he expends 1
searching for a job. Searching in period $n$ incurs a time cost $s_n$. Job offers arrive randomly in a given period. $F_{s_n}(\cdot)$ denotes the distribution function of the present value of the lifetime earnings associated with the highest job offer received over the period. $F_{s_n}(\cdot)$ is stochastically increasing in $s_n$, and for any given $s_n$, the searcher knows the distribution of $F_{s_n}$. Additionally, the individual is free to accept a part-time job from a pool of part-time jobs at any time. These jobs are readily available and all pay a wage of $w^P$ and require a commitment of $h^P$ hours per week, both of which are fixed and given exogenously. This analysis considers all payments net of search costs, which are not specifically analyzed here. When he is part-time employed, his search time is $1 - s_n$.

The government sets two vectors: $U = (u^i, N)$ and $U^P = (u^j^p, N^p, r, d)$, which are unemployment compensation for an unemployed individual and unemployment compensation for an unemployed individual who has taken on a part-time job. $U^P$ is determined as a function of the worker’s income from part-time work. When the individual is working part-time, his benefit is $u^j - w^P h^P (1 - r)$, where $j = 1, 2$, differentiating between the full retained earnings schedule and the partial retained earnings schedule, respectively. The individual chooses $s_n$, the amount of search he commits to in period $n$, $T$, the point at which he accepts the best current full-time job offer, when to take on a part-time job, and $T^P$. If he takes on a part-time job, his
maximum benefit period increases from $N$ to $N'$ because the worker has a fixed pool of benefits from which to draw, $u^jN$, with

$$N' = \begin{cases} 
T^p + u^j(N - T^p) / (u^j - w^p h^p r + d), & \text{if } d < w^p h^p r < u + d \\
T^p + u^j(N - T^p) / (u^j - w^p h^p r), & \text{if } w^p h^p r \leq d 
\end{cases}$$

Where $d$ is the level of the disregard in the full retained earnings payment schedule. If, at the end of $N'$ periods the worker has not accepted a job, he takes on a default job, with earnings $I$. Since part-time work is always available, the lower bound on $I$ is $w^p h^p$.

This paper seeks to compare the two schemes of unemployment compensation described in the Introduction. The first is one in which earnings from a part-time job are disregarded until his income equals $d$, at which point benefits decrease one-for-one with earnings. The second scheme is one in which an additional dollar of earnings decreases benefits by $1 - r$, $0 \leq r \leq 1$. Because the total payments over the course of the spell must be equivalent between the full and partial payment schemes, the following relation holds between the initial unemployment benefit levels:

$$u^2 = \sqrt{u^1(u^1 + 2d)(1 - r)}$$

The individual’s objective function, as suggested by Zuckerman, is given by

$$f(T, T^p, s, \pi | U, U^p) = E[X_{s_n} (T) + Tu^1 + w^p h^p r (T - T^p)]P(T \leq N') + E[I + N'u^1 + w^p h^p r (N' - T^p)]P(T > N')$$

where $X_{s_n} (T)$ is the value of the highest offer received in period $n$, given search level $s_n$. This equation represents the expected lifetime earnings of the individual as a
function of the unemployment benefit and earnings from part-time and full-time employment.

Given that the individual follows the behavior \((T, T^p, s_n)\), the (benevolent) government’s objective is to select \((U = (u, N), U^P = (u^P, N', r, d))\), which maximizes the individual’s expected lifetime income (productivity) net of transfers,

\[
g(U, U^P | T, T^p, \pi, s_n) = E[X_s (T) + w^p h^p r (T - T^p)] P(T \leq N') + E[I + w^p h^p r (N' - T^p)] P(T > N')
\] (2)

Where the payments \(u^i\) do not appear in \(g\) because they represent zero sum transfer payments from the point of view of society\(^2\).

As in Zuckerman, the solution of the model involves a Stackelberg (1952) solution concept, with the government serving as the leader and the worker as the follower. The government anticipates the worker’s reaction \((T^*, T^p^*, s_n^*)\) and therefore determines an optimal policy maximizing \(g(U, U^P | T^*, T^p^*, s_n^*)\) with respect to \(U\) and \(U^P\).

As is common in the literature (Devine 1991), the worker employs a reservation wage strategy, which is to accept the highest full-time wage in a given period, given that it is at least as big as his reservation wage. In this model, Zuckerman defines the reservation wage as the recursive equation

\[
\xi_{n-1} = u^i + w^p h^p r + \xi_n + G_s (\xi_n)
\] (3)

---

\(^1\) As will be shown below, there is only a single spell of part-time work within the unemployment spell.

\(^2\) These payments come from taxpayers and are transferred to the unemployed. So from the point of view of society, any good given to an unemployed worker is taken from an employed worker, and \(g\) seeks to capture the added effect of this transfer. Presumably, these unemployed workers have paid into the system and are therefore receiving their own money, but this model does not take budget balancing into account.
with $\xi_n = I$ and $G_{s_n}(\xi_n) = \int_{\xi_n}^{\infty} (y - \xi_n) dF_s(y)$, where $G_{s_n}$ is the expected return from $s_n$ time units devoted to search in a period, given reservation wage $\xi_n$. It is clear that $G_{s_n}$ is convex, nonnegative, monotonically increasing in $\xi_n$ for any given search effort, and $G_{s_n}(\infty) = 0$.

Proposition 1. The reservation wages $\xi_n$ are decreasing over time.

Proof. The relation in (3), combined with the fact that $G_{s_n}(\xi_n) \geq 0$, establishes the proof of Proposition 1.

III. General Solution

The worker chooses whether or not to work part-time in any given period according to the following rule illustrated in Figure 2 in the Appendix. The worker chooses to search full-time until $T^P$, where his $G_{s_n}$ curve, which represents the value of accepting a full-time job at his reservation wage in period $n$, falls to the point where it equals the value of accepting a part-time job, at which point he chooses to work part-time. When he takes on a part-time job, his $G_{s_n}$ curve will then flatten because the reservation wages increase and the unemployment benefit decreases, leading to an increase in the number of periods available to receive benefits. This is summarized in Proposition 2.
Proposition 2. A worker takes on a part-time job at $G_1 = w^p h^r r + u^l$ and quits the job at $G_{1-h^r} = w^p h^p$.

Proof. For $w^p h^p \leq G_1, G_{1-h^r} \leq w^p h^r r + u^l$ , $G_1(\xi_n) \leq G_{1-h^r}(\xi_n)$, since $G_{\xi_n}$ is monotonically decreasing.

In order to determine what happens when the government changes its policy from full to partial retained earnings, define $D = u^2 + w^p h^r r - (u^1 + w^p h^p)$ as the additional income that the worker receives from a partial retained earnings policy. In order to determine the effect of changing $r$, take $u^1$ and $d$ as fixed, and recalling that $u^2 = \sqrt{u^1(u^1+2d)(1-r)}$, we obtain

$$dD/dr = w^p h^r - \sqrt{(u^1(u^1+2d)/(1-r))}$$

(4)

$$d^2D/dr^2 = -(1/2)\sqrt{u^1(u^1+2d)/(1-r)^3}$$

(5)

Note that $dD/dr > 0$ when $w^p h^p > \sqrt{u^1(u^1+2d)/(1-r)}$ and $d^2D/dr^2 < 0$, implying that $D$ is concave in $r$. So moving from full to partial retained earnings only increases the value of part-time work when $w^p h^p < \sqrt{u^1(u^1+2d)/(1-r)}$. Because of the income cutoff in the full retained earnings schedule, $d + u^1 > w^p h^p > \sqrt{u^1(u^1+2d)/(1-r)}$.

Now consider an increase in the value of part-time work, demonstrated in Figure 3 of the Appendix, either from an increase in $u$ or a decrease in $r$ that increases the total value, as illustrated above. An increase in $w^p h^r r + u$ will also increase $G$, since an
increase in the benefit increases the reservation wage, shifting both the part-time income curve and the $G$ curve up, and also because an increase in $u$ necessarily decreases the number of periods available.

Proposition 3. An increase in the unemployment benefit $u^j$ will increase $T^p$.

Proof. It is clear that since \( \frac{dG_s}{d\xi_n} \geq \frac{dG}{d\xi_n} \), $G$ intersects $w^p h^p r + u^j$ at a larger value of $\xi_n$ than $G$ intersects $w^p h^p r + u^j$.

Now, knowing the individual’s reaction curve, the government employs an optimal policy. In optimizing the government’s objective function, note that while $u^j$ does not directly appear in the function, $N$ is determined by $u^j$, so an optimal $u^{j*}$ and an optimal $N^*$ are complementary. Since $s_n$ and $\pi$ are determined by $T^p$, the independent decisions that the individual makes are $T(U,U^p), T^p(U,U^p)$. We then plug these optimal values back into the government’s objective function.

IV. Numerical Example

Suppose $u = 20$, $d = 10$, $w^p h^p = 20$, $r = 1$, and $F$ is uniformly distributed over the interval $(20,20 + 20s)$, so we have

\[
F = \begin{cases} 
1 / (20s), & 20 \leq y \leq 20 + 20s \\
0, & \text{otherwise}
\end{cases}
\]

And this gives us
We set \( N = 5 \) and calculate the reservation wages and values of \( G_1 \) in each period.

\[
G_1(x) = (1/40)(x^2 - 80x + 1600) \\
G_{1/2}(x) = (1/20)(x^2 - 60x + 900)
\]

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Res Wage</td>
<td>294</td>
<td>118</td>
<td>72.5</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>( G_1 )</td>
<td>152</td>
<td>26</td>
<td>2.5</td>
<td>10</td>
<td>--</td>
</tr>
</tbody>
</table>

So we see that in Period 2, \( G_1 \) falls below the value of the part-time job, giving us \( T^p \).

From here we can calculate \( N' = 8 \), which allows us to calculate the \( G_{1/2} \) curve from here, giving us

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Period} & 1   & 2   & 3   & 4   & 5   & 6   & 7   & 8   \\
\hline
\text{Res Wage} & 1.7E+28 & 1.8E+14 & 1.93E+08 & 62165 & 1134 & 166 & 65 & 20 \\
\hline
\text{G1/2} & 1.40E+57 & 1.6E+29 & 1.93E+08 & 61051 & 924 & 61 & 5 & -- \\
\hline
\end{array}
\]

Note here that \( N' = 7 \), making total time working part-time \( \pi = 5 \).

Now increase the benefit to 30.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Period} & 1   & 2   & 3   & 4   & 5   \\
\hline
\text{Res Wage} & 1060 & 220 & 100 & 60 & 20 \\
\hline
\text{G1} & 810 & 90 & \textbf{10} & 10 & -- \\
\hline
\end{array}
\]

Here, \( T^p = 3 \), \( N' = 6 \). Now compute the part-time reservation wages from \( N' \).

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Period} & 1   & 2   & 3   & 4   & 5   & 6   \\
\hline
\text{Res Wage} & 2834603718 & 238120 & 2201 & 226 & 75 & 20 \\
\hline
\text{G1/2} & 2.80E+09 & 235662 & 1920 & 101 & \textbf{5} & -- \\
\hline
\end{array}
\]

This gives us \( N' = 5 \), \( \pi = 2 \).

There are a couple things this example highlights. Note that the values of \( G \) are increased when the individual is working part-time. So we can see that increasing the
benefit level actually makes part-time work less appealing, which is a surprising result but makes sense in the context of the effects of the unemployment benefit on the reservation wages.

V. Conclusions

A simple model that adapts Zuckerman’s search model has been presented with the introduction of subsidized part-time work. Conditions for the worker’s optimal full-time/part-time/unemployment decision have been presented. There are, however, a number of issues that need mention.

This model does not assign a value to leisure. A more complex model could take into account the tradeoff between consumption and leisure. This would require relaxing the assumption that the worker’s time is inelastically set to be for either search or work, which is not too philosophically difficult to allow. Another area it would be illuminating to explore is in other shapes of $G$. This analysis took it to be uniform for mathematical simplicity, but experimenting with normal distributions or otherwise may introduce other complexities to the analysis.

This analysis takes all unemployment benefits to be transfer payments with a zero-sum effect on society. An extension of this analysis would view this in a different light, by either taking past employment into consideration when determining benefit levels or taking into consideration total social welfare. In either case, considering a balanced budget would be beneficial for further analysis.

More importantly, the result that an increase in the value of part-time work leads to a reduction in part-time work needs further exploration for being counterintuitive and
contrary to the literature. McCall’s (1997) results demonstrated that an increase in the disregard leads to an initial increase in working part-time, but a decrease over time in the value of part-time work because of the decreasing reservation wages. Also, while this paper does explore what happens when the part-time work replacement rate changes, and suggests that states with partial retained earnings will dissuade unemployed workers from finding part-time jobs, more work needs to be done in the future to determine whether or not these systems are any better than full retained earnings.

References


Figure 1. Full retained earnings schedule versus partial retained earnings
Figure 2. When the part-time income curve is greater than the G curve, the worker takes on a part-time job, resulting in a flatter G curve, where he continues to work until his part-time G curve is equal to I.
Figure 3. An increase in $u$ leads to a delay in the acceptance of a part-time job and decreases the maximum length of the eligibility period.