Outline

- Registration Background
- Medial Scaffold: Representation for 3D Shapes
- Graduated Assignment Graph Matching
- Results
- Conclusions
Registration: Defining Correspondence

- **Fundamental** for processing scanned objects, modeling, matching, recognition, medical applications, etc.

- **Local Registration**
  Initial position given. ICP and its improvements
  Survey: [Campbell & Flynn CVIU’01], [3DIM’03]

- **Global Registration**
  Skeleton-based, Surface-feature based

More difficult. Main focus of this talk.
Local Registration: Iterative Closest Points (ICP)

- [Besl & McKay PAMI’92]
- Needs a good initial alignment
- Local search problems
  - Sensitive to local minimum, noise
  - May converge slowly
  - Lack of surface representation
- Improvements:
  - [Chen & Mendiono] accuracy: match closest point on the projected plane
  - Use color, non-rigid match to get better convergence, etc.
Global Registration

- **Surface featured based**
  - [Wyngaerd & Van Gool CVIU’02]: bitangent curve pairs as surface landmarks
  - [Allen et. al.’03]: straight lines as features in aligning architectural dataset

- **Skeletal graph based**
  - [Brennecke & Isenberg ’04]:
    - Internal skeletal graph of a closed surface mesh, using an edge collapse algorithm
    - match largest common subgraph
  - [Sundar et. al. ’03]: Skeletal tree from thinning voxels via a distance transform, coarse-to-fine matching

1. Skeletons over-simplified
2. Graph topology not handled well
Proposed: Match the Medial Scaffold

- **Medial Scaffold**: medial structure in the form of a 3D hypergraph
Medial Scaffold

- Blum’s medial axis (grassfire), wave propagation

- 3D: Five types of points [Giblin & Kimia PAMI’04]:
  - Sheet: $A_1^2$
  - Links: $A_1^3$ (Axial), $A_3$ (Rib)
  - Nodes: $A_1^4$, $A_1A_3$

$A_k^n$: contact at $n$ distinct points, each with $k+1$ degree of contact
Compute the Medial Scaffold

3D [Leymarie PhD]: Medial Scaffold Detection + Segregation

Point Cloud → Propagation → Full Shock Scaffold → Segregation

- Sampling Artifact Scaffold
- Surface Scaffold
- Medial Scaffold
- Full Scaffold
- Surface Scaffold
Medial Structure Hierarchy

- Medial Axis ($MA$)
- Shock Hypergraph ($SH$)
- Shock Scaffold with Sheets ($SC^+$)
- Shock Scaffold ($SC$)

Only need to detect special nodes and links, while maintaining their connectivity.
Medial Structure Regularization

- Medial Axis is sensitive to noise & perturbations.
- **Transitions**: sudden changes in topology
- 2D examples:

  The growth of an axis with small perturbations ($A_1A_3$)

  The swapping of MA branches

  Pruning:

  Smoothing/medial branch pruning
Seven Types of Transitions in 3D

[Giblin & Kimia ECCV’02]
Scaffold Regularization [Leymarie et. al. ICPR’04]

- Transition removal, i.e. remove topological instability
- Smoothing

Blue: $A_3$ links, Red: $A_1^4$ links

Green: $A_1A_3$ nodes, Pink: $A_1^4$ nodes

$Leymarie$ et. al. $ICPR’04$
Match Medial Scaffolds by Graph Matching

- **Intractability**
  - Weighted graph matching: \textbf{NP}-hard
  - One special case: Largest common subgraph: \textbf{NP}-complete
  - Only “good” \textit{sub-optimal} solutions can be found

- **Graduated Assignment** [Gold & Rangarajan PAMI’96]
  - [Sharvit \textit{et. al.} JVCIR’98] index 25-shape database by matching 2D \textit{shock graphs}

- 3D \textbf{hypergraph} matching:
  - Additional dimension
  - Generally not a \textit{tree}, might have isolated \textit{loops}
  - No inside/outside: non-closed surfaces or surface patches
**Quadratic weighted graph matching**

G, \( \bar{G} \): 2 undirected graphs
I: # of nodes in G, \( \bar{I} \): # of nodes in \( \bar{G} \)
{\( G_i \)}, \{\( \bar{G}_i \)\} nodes
{\( G_{ij} \)}, \{\( \bar{G}_{ij} \)\} edges: adjacency matrices of graphs

The **match matrix**

\[ M_{ii} = \begin{cases} 1 & \text{if node } i \text{ in } G \text{ corresponds to node } \bar{i} \text{ in } \bar{G}, \\ 0 & \text{otherwise} \end{cases} \]

Then objective function to maximize over the space of \( M \) is:

\[
E(M) = \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} \sum_{j=1}^{I} \sum_{\bar{j}=1}^{\bar{I}} M_{ii}\bar{i}M_{jj}\bar{j}L_{ii\bar{j}\bar{j}} + \alpha \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} M_{ii}\bar{i}N_{ii\bar{i}}
\]

\( L_{ii\bar{j}\bar{j}} \): link similarity between \( G_{ij} \) and \( \bar{G}_{\bar{i}\bar{j}} \)
\( N_{ii\bar{i}} \): node similarity between \( G_i \) and \( \bar{G}_i \)

Cost of matching \( G_{ij} \) to \( \bar{G}_{ij} \):
If the nodes match, how similar the links are.

Cost of matching \( G_i \) to \( \bar{G}_i \)
Modified Graduated Assignment for 3D Medial Scaffold Matching

\[ E(M) = \alpha \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} M_{\bar{i}i} N_{\bar{i}i} + \beta \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} \sum_{j=1}^{I} \sum_{\bar{j}=1}^{\bar{I}} M_{\bar{i}i} M_{\bar{j}j} L_{\bar{i}i\bar{j}j} + \sum_{i=1}^{I} \sum_{\bar{i}=1}^{\bar{I}} \sum_{j=1}^{I} \sum_{\bar{j}=1}^{\bar{I}} \sum_{k=1}^{I} \sum_{\bar{k}=1}^{\bar{I}} M_{\bar{i}i} M_{\bar{j}j} M_{k\bar{k}} H_{\bar{i}i\bar{j}j\bar{k}\bar{k}}, \]

\( \alpha, \beta \) weights

Node cost: (radius)

\[ N_{\bar{i}i}(G_i, \overline{G_i}) = \begin{cases} 0, & \text{if } G_i \text{ and } \overline{G_i} \text{ have different types,} \\ 1 - \left| \frac{r_i - r_{\bar{i}}}{\max(R, R)} \right|, & \text{otherwise,} \end{cases} \]

Link cost: (length)

\[ L_{\bar{i}i\bar{j}j} = \begin{cases} 0, & \text{if any of links } ij \text{ and } \overline{ij} \text{ are missing,} \\ 1 - \left| \frac{l_{ij} - l_{\overline{ij}}}{\max(L, L)} \right|, & \text{otherwise,} \end{cases} \]

Sheet (hyperlink) cost: (corner angle)

\[ H_{\bar{i}i\bar{j}j\bar{k}k} = \begin{cases} 0, & \text{if any links } ij, jk, \overline{ij}, \overline{jk} \text{ are missing,} \\ 1 - | \angle ijk - \angle \overline{ijk} |, & \text{otherwise,} \end{cases} \]
Results: Sheep

Sheep 20K points, after surface reconstruction
The scaffold matching is good enough that ICP is not required.

Result of Scaffold Graph Matching

Two scans of an object at the same resolution (20K points):

Colors to represent correct link matches; grays to represent miss matches.
Results: David Head

Two sub-samples from the ground truth (42350 points)
Matching Results

Scaffold matching result

Scaffold matching + ICP

Validation against the ground truth: (object dimension = 69x69x76)
average sq dist 3.129372

average sq dist 0.000005
Partial Shape Matching: Sheep with the rear portion cut off

Sheep 1-20K scaffold

Sheep 1-20K with the rear portion cut
Partial Shape Matching (2\textsuperscript{nd} example)

Sheep (2K points)

Another sheep of 2K points, but with no samples on the bottom
Partial Shape Matching (cont’d)

No match & Incorrect matches! Global registration still succeeds.

Result of scaffold matching

Result after ICP
Two scans of the outside surface of a pot (50K and 40K). The inner surface of the pot is missing.
The Full Scaffold

Both the inside and outside medial structures are connected together via shock sheets.
Alignment by Scaffold Matching

The scaffold matching result
Final Registration after ICP
Two Possible Reasons for Incorrect Matches

- Graduated assignment matching is not optimal.

Typically this does not affect the overall registration if a sufficient number of nodes are correctly assigned.
Reasons for Incorrect Matches (cont’d)

- Medial structure transitions are not completely handled.

1. Only 8 shock vertices to match
2. Transitions not completely handled

Result of shock matching

MIS-MATCH!!
Benefits of 3D Medial Scaffolds

- A global **hierarchical** structure is built-in.
- **Scale** is represented.
- Salient features are captured:
  - Generalized axes of elongated objects
  - *curvature extrema* and **ridges**
- The medial representation is **complete**.
- **Reconstruction** of the shape is always possible.
- **Robust** after regularization.
- **Easy to handle** shape **deformations**.

Data from Cyberware Inc.
Conclusions

Global Registration by Matching Medial Scaffolds

- Take input as point clouds or partial meshes.
- Robust to noise. Invariant under different resolutions & acquisition conditions.

- **Skeleton:**
  - Can be *graphs* with loops (not a *tree*).
  - Contains *sheets, links, nodes*. Not over-simplified.
  - Carefully Regularized.

- **Match:**
  - Nearly-optimal.
  - Can be improved to do *fine* registration.
  - Can be extended to register *non-rigid* objects.
  - Can be extended to do *recognition*. 
Thank You

Acknowledgements

- This material is based upon work supported by the National Science Foundation under Grants 0205477 and 0083231.