Specific notation allows network concepts (actors, relations between actors, attributes of actors) to be referred to in a unified manner.

**Graph Theoretic Notation** (p. 71)
- Views the network as a graph, consisting of nodes joined by lines.
- “N” refers to a set with “g” number of actors (there can be g(g-1) actors, 0 possible).
- “The symbol N is commonly used to stand for the set, since graph theory literature frequently refers to this set as a collection of nodes of a graph” (p. 71).
- If you have an ordered pair of actors, n(i) and n(j), in which n(i) relates to n(j), then the pair is an element of a special collection of pairs referred to as “L”.
- “We use the symbol L to refer to the set of directed Lines and the symbol l to refer to the individual directed lines in the set” (p. 72).

**Directional Relations:**

\[
\begin{align*}
N &= \{\text{Allison, Drew, Eliot, Keith, Ross, Sarah}\} \quad \text{[the set]} \\
g &= 6 \text{ second grade children} \quad \text{[number of actors]} \\
n(1) &= \text{Allison, n(2) = Drew, n(3) = Eliot, etc.} \quad \text{[symbols for the actors]} \\
n(1) \text{ relates to n(2)} \quad \text{[ordered pair or element]} \\
L &= \{l(1), l(2), \ldots l(L)\} \quad \text{[set of directed lines]} \\
l(1) &= n(1) \to n(2) \quad \text{[a single ordered pair in L, refers to the individual directed line or arc]} \\
G &= (N, L) \quad \text{[graph = set of nodes, set of lines]} \\
\end{align*}
\]

**Non-Directional Relations (ex. Friendship)**

\[
\begin{align*}
g(g-1) &= 6(6-1) = 30 \quad \text{[number of possible ordered pairs]} \\
L &= 8 \quad \text{[8 friendships out of the 30 ordered pairs]} \\
L &= l(1) = \langle\text{Allison, Ross}\rangle, l(2), \text{etc.} \quad \text{[Allison and Ross are an ordered pair or element]} \\
l(1) &= \langle\text{Allison, Ross}\rangle \quad \text{[Allison views Ross as a friend]} \\
\end{align*}
\]
As long as Ross views Allison as a friend, the relation is nondirectional

**Multiple Relations:**

R \[\text{Total number of relations in } N\]

R = 3 \[\text{types of relations: friends at the beginning of the year, friends at the end of the year, lives near}\]

L(1) = 8 \[8 \text{ pairs stated they were friends at the beginning of the year}\]

<Allison, Ross> \[<> \text{ ties on a directional relation}\]

(Sarah, Drew) \[() \text{ ties on a nondirectional relation, each pair can only be listed once}\]

- “Graph theory is not well designed for data sets that record the strength or frequency of the interaction for a pair of actors” (p. 76).

**Sociometric Notation**

- Study of positive and negative affective relations (friends/enemies).
- Sociometric: a social network data set consisting of people, measuring affective relations.
- Sociomatries: relational data presented in two-way matrices, (rows = sending actors, columns = receiving actors).
- “A sociomatrix for a dichotomous relation is exactly the adjacency matrix for the graph (or sociogram) quantifying the ties between the actors for the relation in question” (p. 77).
- The relation can take on any value from 0 to C-1.
- The diagonals of the sociomatries are undefined (i.e., you cannot choose yourself as a friend).

**Single Relation:**

X \[\text{single valued, directional relation}\]

measured on the ordered pairs of actors that can be formed from the actors in N

X \[\text{Associated sociomatrix with the}\]
value of the tie from \( n(i) \) to \( n(j) \) placed into the \((i,j)\)th element of \( X\]

\[ x(i)(j) \]

[value of the tie from \( n(i) \) to \( n(j) \) on relation \( x \)]

\[ n(i), n(i) \]

[pairs listing the same actor twice, “self-choices” for a specific relation where \( i = 1,2\ldots g \)]

\[ C \]

[Number of different values the tie can take on]

**Multiple Relations:**

\[ R = 3 \]

[Number of relations measured on a single set of actors…continued from above example]

\[ X(r) \]

[the sociomatrix associated with relation \( x \) \( (r) \)]

\[ x(i)(j)(r) \]

[the value of the tie from \( n(i) \) to \( n(j) \) on relation \( X \) \( (r) \)]

See Table 3.1, pg. 82

\[ g \times g \times R \]

[Super-sociomatrix: represents the actors and relations in a multirelational network]

**Algebraic Notations**

- “Most useful for multirelational networks since it easily denotes the “combinations” of relations in these networks” (p. 84).
- Notation scheme cannot handle valued relations or actor attributes.

\[ x(i)(F)(j) = 1 \]

[n(i) is a Friend of n(j), implies that there is a “1” in the cell at row i and column j of the sociomatrix for this relation]

**Two Sets of Actors:**

First Actor = sender, \( N \) = first set of actors, \( g \) = number of actors
Second Actors = receiver, \( M \) = first set of receivers, \( h \) = number of actors

There are \( h/2 \) dyads that can be formed from actors in \( M \).
Different Types of Pairs:

**Homogeneous**: pairs that consist of actors from the same sets.
- Sender and Receiver both belong to \( N \).
- Sender and Receiver both belong to \( M \).

**Heterogeneous**: pairs that consist of actors from different sets.
- Sender belongs to \( N \) and Receiver belongs to \( M \).
- Sender belongs to \( M \) and Receiver belongs to \( N \).

Sociometric Notation:
The \((i,j)\)th entry of the sociomatrix \( X(r) \) is defined as:
\[
x(i)(j)(r) = \text{the value of the tie from } n(i) \text{ to } m(j) \text{ on the relation } X \ (r)
\]

**Wasserman and Faust, Chapter 4: Graphs and Matrices**

**Why Graphs?**
- Provides a vocabulary which can be used to label and denote many social structural properties.
- Gives us mathematical operations and ideas with which many of these properties can be quantified and measured.
- Gives us the ability to prove theorems about graphs.
- Gives us a representation of a social network as a model of a social system.

**GRAPHS**
**Graph**: a model for a social network with an undirected dichotomous relation.
**Adjacent**: \( l(k) = [n(i), n(j)] \), \( i \) and \( j \) are said to be adjacent.
**Incident**: a node is said to be incident with a line if its one of the unordered pair of nodes defining the line.
**Trivial**: a graph containing only one node.
**Empty**: a graph containing nodes but no lines.

**Subgraphs**
**Subgraph**: \( G(s) \) is a subgraph of \( G \) if the set of nodes/lines of \( G(s) \) is a subset of the set of nodes/lines of \( G \).
**Dyads**: a pair of actors and the possible tie between them.
**Triads**: consists of three nodes and the possible lines between them (0-3).
- Forbidden triad (Granovetter, 1973): only two lines, unlikely that there is no relation between the nodes without a line.
**Degree of a node** [\( d(n(i)) \)]: a measure of activity of the actor it represents, can be found by counting the number of lines incident with it (pg. 100 for equation).
- \( d \)-regular: all the degrees of all the nodes are equal.
**Density**: the proportion of possible lines that are actually present in the graph (0 to 1) (pg. 101 for equation).
**Degree**: the number of lines incident with each node in a graph.
- Complete graph: all lines are present, all nodes are adjacent
It is often important to know whether it is possible to reach some node \( n(i) \) from another node \( n(j) \), to know how many ways it can be done, and which of these ways is optimal according to some criteria.

**Walk** \( (W) \): sequence of nodes and lines in which each node is incident with the lines following and preceding it in the sequence (lines can be counted more than once).

**Length of a walk** \( (l) \): number of occurrences of lines in it.

**Origin of a walk**: node that the walk begins with.

**Terminus of a walk**: node that the walk ends with.

**Inverse of a walk** \( (W^{-1}) \): walk listed in exactly the opposite order.

**Trail**: a walk in which all of the lines are distinct, though a node can be included more than once.

**Length of a trail**: number of lines in the trail.

**Path**: a walk in which all nodes and all lines are distinct.

**Reachable**: when there is a path between two nodes, the nodes are said to be reachable.

**Closed walk**: begins and ends at the same node.

**Cycle**: closed walk of at least three nodes in which all lines are distinct, and all nodes except the beginning and end are distinct.

**Semi-cycle**: closed sequence of distinct nodes and arcs in which each node is either adjacent to or adjacent from the previous node in the sequence.

**Tour**: closed walk in which each line in the graph is used at least once, important in terms of balance and clusterability in signed graphs.

**Eulerian trails**: closed trails that include every line exactly once.

**Hamiltonian cycle**: every node in the graph is included exactly once.

**Connected graph**: there is a path between every pair of nodes in the graph (if not, its disconnected).

**Component**: is a subgraph in which there is a path between all pairs of nodes in the subgraph (and there is no path between a node in the component and any node not in the component).

**Geodesic**: shortest path between two nodes

“Distances are quite important in social network analyses. They quantify how far apart each pair of nodes is, and are used in two of the centrality measures and are important consideration for constructing some kinds of cohesive subgroups” (p. 111)

**Eccentricity or association number** of a node is the largest geodesic distance between that node and any other node.

**Diameter**: the length of the largest geodesic between any pair of nodes.

**Cutpoints**: a node where the number of components in the graph that contains the node is fewer than the number of components in the subgraph that results from deleting the node from the graph.

**Bridge**: is a line such that the graph containing the line has fewer components than the subgraph that is obtained after the line is removed.

**l-line cut**: is a set of \( l \) lines that, if deleted, disconnects the graph.

**Cohesive graphs** have many short geodesics and small diameters.

**Vulnerable graphs** are more likely to become disconnected if a few nodes or lines are removed.
**Point-connectivity or node-connectivity** \([k(G)]\): minimum number of nodes \(k\) that must be removed to make the graph disconnected or to leave a trivial graph.

**Line-connectivity or edge-connectivity** \([\lambda(G)]\): minimum number of lines \(\lambda\) that must be removed to disconnect the graph or leave a trivial graph.

**Isomorphic**: two graphs \(G\) and \(G^*\) with a one-to-one mapping from the nodes of \(G\) to the nodes of \(G^*\) that preserves the adjacency of the nodes (will be identical on all graph theoretic properties).

**Complement**: a graph which has the same set of nodes as another graph.

**Tree**: a graph that is connected and contains no cycles.

**Forest**: a graph that is disconnected and contains no cycles.

**Bipartite Graphs**: nodes of a graph can be partitioned into two subsets, in which every line has an unordered pair of nodes with a node in each subset (complete bipartite graph has every node in subset one adjacent to every node in subset two).

**DIRECTED GRAPHS**

**Directional**: ties are oriented from one actor to another.

**Directed graph or digraph**: consists of two sets of information: a set of nodes and a set of arcs (ordered pairs).

Isomorphism classes of dyads:

- **Null dyad**: no arcs between two nodes.
- **Asymmetric dyad**: arc between the two nodes going in one direction.
- **Mutual or reciprocal dyad**: two arcs between the nodes, one going in one direction and one in the opposite direction.

**Indegree of a node**: number of nodes that are adjacent to n(i).

**Outdegree of a node**: number of nodes adjacent from n(i).

See pg. 128 for explanation of Isolate, Transmitter, Receiver, Carrier or Ordinary.

**Directed walk**: sequence of alternating nodes and arcs so that each arc has its origin at the previous node and its terminus at the subsequent node.

**Directed trail**: directed walk in which no arc is included more than once.

**Directed path**: directed walk in which no node and no arc is included more than once.

**Reachability**: there is a direct path from n(i) to n(j) \([j] \text{ is reachable from } n(i)\].

**Connectivity by path or semipath**: Weakly, Unilaterally, Strongly, Recursively (p. 132).

**Connectivity for digraph**: Weakly, Unilaterally, Strongly, Recursively (p. 133).

**Converse**: reversing the direction of all arcs.

**Tournaments**: represents a set of actors competing in some event and a relation indicating superior performances in competition.

**SIGNED GRAPHS** (vocabulary is the same for graphs except where noted)

**Signed**: lines carry a positive or negative sign (valence).

**VALUED GRAPHS** (vocabulary is the same for graphs except where noted)

**Valued**: the strength or intensity of each line or arc is recorded [can refer to directional (dollar amount of exports) and nondirectional (number of interactions) data].

**Integer weighted digraph**: all values in the valued digraph are from the set of integers.
Markov chains: set of graphs whose values are probabilities.  
Value of a path: is equal to the smallest value attached to any line in it.  
Reachability: level is set at the strongest line between the nodes.  
Path length: sum of the values of the lines in it.

MULTIGRAPHS  
Multigraph or multivariate graph: allows more than one relation or set of lines.  
“If more than one relation is measured on the same set of actors, then the graph representing this network must allow each pair of nodes to be connected in more than one way” (p. 146).

HYPERGRAPHS  
Appropriate for membership or affiliation networks.  
Hypergraph: consists of a set of objects and a collection of subsets of objects, in which each object belongs to at least on subset, and no subset is empty.  
Points: objects.  
Edges: collections of objects.

RELATIONS  
Relations: ordered pairs of actors in a network between whom a substantive tie is present.  
Reflexivity: all possible ties are present in a relation.  
Symmetric: all the dyads are either mutual or null.  
Transitive: essentially, if j chooses k as a friend, and k chooses l, then j will choose l as a friend.

MATRICES  
Sociomatrix (X): record which pairs of nodes are adjacent (contains only 1’s and 0’s for a graph, and is symmetric for a nondirectional relation).  
Incident matrix (I): records which lines are incident with which nodes (nodes index rows, lines columns).  
Martices for hypergraphs: codes which points are incident with which edges.  
Size (order): the number of rows and columns in the matrix.  
Cell: each entry.  
Main Diagonal: entries are “self-choices” or loops, may be undefined.  
Matrix Permutation: any reordering of the objects (helpful in seeing patterns).  
Transpose: interchanging the rows and columns (change direction of ties between actors).  
Addition: sum of the elements in the corresponding cells (came size matrices).  
Subtraction: difference between elements.  
Multiplication: see p. 157.  
Powers of a matrix: the matrix times itself.  
Boolean: the multiplication of matrices is referred to as having a value of 1 or 0.