12 Network Positions and Roles

12.1 – Background

This chapter is primarily concerned with defining equivalences and measuring how closely subsets of actors adhere to these definitions.

Individual roles are descriptions of the network, including similarities among actors, and associations among relations, from the perspectives of individual actors.

This chapter:

• reviews theoretical background for positions and roles in social networks
• discusses different levels of role analysis of social networks
• describes and illustrates different approaches for defining equivalence of actors, and as appropriate, for measuring the degree of equivalence
• concludes with a comparison of the different approaches.

12.1.1 – Theoretical Definitions of Roles and Positions

In contrast to social position, which refers to a collection of actors, the concept of social role refers to the ways in which occupants of a position relate to occupants of other positions.

Theoretical definitions of social role often are stated as properties of individuals or sets of individuals.

Many authors had not made the distinction between status and position.

Social identity is the aggregate of its composite statuses. A particular social status involves, not a single associated role, but an array of associated roles.

To translate theoretical notions of role and position into empirical social network methods, one assumes that the measured relations in a set of social network data are indicators of the roles of actors in different social positions.

The goal of an individual role analysis is to describe the regularities in the ties that link an actor (ego) to other actors (alters).

Social network methods for studying individual roles focus on patterns or “types” of ties among actors or subsets of actors as a way to formalize the notion of social role.

12.1.2 – Levels of Role Analysis in Social Networks

In network analysis, the notion of role has been used at three conceptual levels: the entire group, a subset of actors, and an individual actor.

• Global role structures describe an entire group.
• Whereas role structures pertain to an entire group, local roles pertain to subsets of actors within a group. The subsets can be positions of equivalent actors (for example subsets of approximately structurally equivalent actors).
• At the most specific level of analysis, roles can pertain to individual actors. These are referred to as individual or ego roles. Analyses at this level study the patterns and regularities in ties from the perspective of individual actors.

12.1.3 – Equivalences in Networks
Each approach to network roles or positions specifies the graph theoretic or network properties that sets of actors must have in order to be considered equivalent in terms of the roles they play and thus the positions they occupy.

Actors who are equivalent are assigned to the same equivalence class or position.

Each equivalence definition specifies an equivalence relation, which we denote by $\equiv$.

An equivalence relation has three important properties:
• it is symmetric ($i \equiv j$ if and only if $j \equiv i$)
• reflexive ($i \equiv i$)
• transitive (if $i \equiv j$ and $j \equiv k$ then $i \equiv k$).

This chapter discusses several of the more general approaches, including:
• Automorphic and isomorphic equivalence
• Regular equivalence
• Local role equivalence
• Ego algebras

12.2 – Structural Equivalence Revisited
Two actors are structurally equivalent if and only if they have identical ties to and from identical other actors.

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Since structural equivalence requires identical ties to and from identical other actors, in this example people can only be structurally equivalent if they supervise exactly the same other people, and are supervised by exactly the same others.

The fact that structurally equivalent actors must have identical ties to and from identical other actors is a severe limitation.

Two actors can be assigned to the same “manager” position only if they supervise exactly the same employees. Managers from two different companies, or even managers in charge of two different cannot be structurally equivalent.

12.3 – Automorphic and Isomorphic Equivalence

12.3.1 – Definition
Two graphs or directed graphs are isomorphic if there is a one-to-one mapping of the nodes in one graph to the nodes in the other graph that preserves the property of adjacency.

The property of isomorphism maps one graph (or directed graph) to another graph (or directed graph).

An analogous idea, called an automorphism, is defined for a single graph (or directed graph).

If the mapping, τ, is from the nodes in a graph (or directed graph) back to themselves (rather than from one graph to another), then the mapping is called an automorphism.

12.3.2 – Example
The term “automorphism” refers to a mapping of a graph (or directed graph) onto itself, whereas the term “isomorphism” refers to the mapping of one graph (or directed graph) onto another.

Two nodes that are automorphically equivalent have the same indegree (ties to the node from other nodes), the same outdegree (ties from the node to other nodes), the same centrality on every possible measure (for example, betweenness, closeness, etc.), belong to the same number and size of cliques, and so on.

We can define isomorphic equivalence in terms of a one-to-one mapping of nodes in one graph (or directed graph) to nodes in another graph (or directed graph).
An orbit refers to a subset of nodes in a graph (or directed graph) that can be mapped to one another.

Structurally equivalent
- 1
- 2
- 3
- 4
- 5, 6
- 7
- 8, 9

Automorphically related
- 2, 4
- 5, 6, 7, 9

12.3.3 – Measuring Automorphic Equivalence

One of the limitations of automorphic and isomorphic equivalence as an approach for analyzing social networks is that there is no known fast algorithm that guarantees identification of automorphically equivalent nodes in all graphs.

For measurement, subsets of automorphically (or isomorphically) equivalent nodes must be contained within subsets of nodes that have identical values on all graph theoretic measures.

In a corporation managers of different size departments would not be automorphically equivalent.

The problem remains, though, of how to measure the degree of automorphic equivalence between pairs of nodes in a way that is not arbitrary, and that is not difficult to compute.

Structurally equivalent
- 1
- 2
- 3
- 4
- 5, 6
- 7
- 8, 9

Automorphically related
- 2, 4
- 5, 6, 7, 8, 9

Not automorphically related
- 2, 3, 4
- 5, 6, 7, 8, 9

The restriction of equal number of ties is relaxed by the notion of regular equivalence.
12.4 – Regular Equivalence

The notion of regular equivalence formalizes the observation that actors who occupy the same social position relate in the same ways with other actors who are themselves in the same positions.

Regular equivalence does not require actors to have identical ties to identical other actors (as required by structural equivalence) or to be structurally indistinguishable (as required by automorphic or isomorphic equivalence).

12.4.1 – Definition of Regular Equivalence

Briefly, actors who are regularly equivalent have identical ties to and from equivalent actors.

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<th>Structurally equivalent</th>
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Notice that these equivalence classes are exactly the three “levels” in the hierarchy, and might correspond to the CEO, managers, and employees in this hypothetical company.
The **coarsest partition** (the partition with the fewest equivalence classes) that is consistent with the definition of regular equivalence is called the maximal regular equivalence.

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<th>Structurally equivalent</th>
<th>Automorphically related</th>
<th>Not automorphically related</th>
<th>Regular equivalence is</th>
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12.4.2 – Regular Equivalence for Nondirectional Relations
As many authors have noted, in a graph (for a nondirectional relation) in which there are **no isolates**, the maximal regular equivalence consists of a **single equivalence class** containing all nodes.

A partition consisting of a single equivalence class is trivial, and probably uninteresting. However, a nondirectional relation may also contain other regular equivalence partitions.

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<th>Regular equivalence is</th>
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<td>• 1,3,4</td>
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The **neighborhood** of a node i in a graph consists of all nodes adjacent to node i.

Briefly, in order to be regularly equivalent, actors must be adjacent to the **same kinds** (equivalence classes) of other actors.
12.4.3 – Regular Equivalence Blockmodels

A blockmodel consists of a mapping of actors into equivalence classes (or positions) according to the particular equivalence definition, and for each pair of positions, a statement of whether or not there is a tie present from one position to another position.

The difference between structural equivalence blockmodels and regular equivalence blockmodels is the rule for determining which blocks are oneblocks (presence of a tie) and which blocks are zeroblocks (absence of a tie), and consequently, what oneblocks and zeroblocks imply about the corresponding entries in the submatrices of the sociomatrix.

12.4.4 – A Measure of Regular Equivalence

The earliest approaches to regular equivalence presented measures of the degree of regular equivalence for pairs of actors in a network.

More recently, authors have focused on methods for assigning actors to subsets such that the partition of actors is optimal in the sense that actors in the same subset are nearly regularly equivalent.

Finding subsets of regularly equivalent actors in a network data set requires simultaneously deciding whether or not the alters to whom potentially regularly equivalent actors are tied are themselves regularly equivalent.

One way to measure how close pairs or subsets of actors are to being regularly equivalent is to consider how well the ties to and from pairs of actors “match” each other.

The REGE algorithm uses an iterative procedure in which estimates of the degree of regular equivalence between pairs of actors are adjusted in light of the equivalences of the alters adjacent to and from members of the pair.

REGE utilizes a function of how well i’s ties to and from all actors can be “matched” by j’s ties to and from all actors, and vice versa.

It provides a quantity that ranges from 0 to 1 (if i and j are perfectly regularly equivalent).

In the computation of Mij, the equivalence of each pair of actors is revised at each iteration, t, in light of the equivalence of other pairs of actors in the network.

In practice one must decide how many iterations of the REGE procedure to run before accepting an estimate of pairwise regular equivalence.

In practice measuring regular equivalence using this algorithm is problematic in many situations.

• First, it is important to note that this equivalence measure counts ties matched between two actors, not the number of alters matched.
• Also, when relations are nondirectional (and there are no isolates), when relations are reflexive (i → i for all i), or when each actor is involved in at least one reciprocated tie (so that for each i
there exists some \( k \) such that \( i \leftrightarrow k \), then maximizing this measure finds the maximal regular equivalence in which all actors are perfectly regularly equivalent.

- In addition, since the algorithm counts ties matched (rather than actors matched), the indegree and outdegree of each actor influence the measure of equivalence.
- Finally, since a given network may contain several regular equivalence partitions, other regular equivalences may exist in the network that are not found by the above algorithm.

12.4.5 – An Example
One can use the program REGE in UCINET to do the calculations. One can use three iterations, and included more than one relation and their transposes in the calculation. A result is a symmetric matrix of similarities (the \( M_{ij} \)). UCINET seems to do this too.

12.5 – "Types" of Ties
Let us consider two definitions of equivalence that focus on the types of ties in which each actor is involved. These two approaches, Winship and Mandel’s local role equivalence and Breiger and Pattison’s ego algebras, consider associations among relations from the perspectives of individual actors.

For a single position, the collection of all of the ways in which an occupant of a particular position relates to others in other positions is called the role set of the position.

If actors are involved in the same types of ties, then they perform the same network role, and are assigned to the same position.

12.5.1 – An Example

<table>
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<tr>
<th>Two primitive relations, labeled H and L, and an additional three compound relations (including the null relation).</th>
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<td>1 ( \rightarrow ) 5: ( S_{1,5} = { HH } )</td>
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<tr>
<td>1 ( \rightarrow ) 6: ( S_{1,6} = { HH } )</td>
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</table>

list describes the types of ties that are defined for actor 1. Each type is a role relation, and the collection of different role relations constitutes actor 1’s role set.

12.6 – Local Role Equivalence
Winship and Mandel (1983) use the role set of each actor to define local role equivalence, or simply role equivalence. Two actors are role equivalent (LRE) if they have identical role sets.
Actors i and j are role equivalent if the collection of ways in which actor i relates to other actors is the same as the collection of ways in which actor j relates to other actors.

12.6.1 – Measuring Local Role Dissimilarity
In actual social network data, it is unlikely that two actors will be perfectly role equivalent. The measure of role equivalence between actors focuses on how well the role relations in two actors’ role sets “match” each other.

A measure of the dissimilarity of two role relations is the city block distance between the vectors that code the role relations in the super-sociomatrix (which is equal to the sum of the absolute value of the differences between corresponding entries).

An important feature of role equivalence is that it can be generalized to measure the role equivalence of actors from different networks, so long as the same relations are measured in both networks. For role equivalence is more general than regular equivalence. Unlike regular equivalence, role equivalence does not require that role equivalent actors be tied by the same role relations to actors who are themselves role equivalent.

12.6.2 – Examples
The authors use the routine WINMAN in the program ROLE, and create a complete link hierarchical clustering using the program SYSTAT to calculate role equivalences for advice and friendship relations for Krackhardt’s high-tech managers. UCINET seems to do this too.

12.7 – Ego Algebras
Breiger and Pattison (1986) present a comprehensive scheme for modeling individual actors’ roles and group role structure simultaneously.

The scheme presented by Breiger and Pattison has two major parts: the first describes the perspectives of the individual actors in order to study which actors have similar roles or positions in a network, and the second summarizes the relational features that are common to all members of the network.

The idea of ego algebras is that an individual’s view of the network is based, in part, on which sets of relations “go together” by always occurring together for that actor.

12.7.1 – Definition of Ego Algebras
A compound relation is the combination of two relations.

Two relations are identical if they have ties between exactly the same pairs of actors.

The relations “is a friend of” and “goes to help and advice” are identical in some network if whenever a person nominates another as a friend, they also name the other person as someone they go to for help and advice, and vice versa.
The operation of composition of relations from the perspective of an individual actor focuses on ties emanating from the actor.

**12.7.2 – Equivalence of Ego Algebras**
Two actors have identical ego algebras, and are thus ego-algebraically equivalent (EA), if the equivalences among relations and the composition of relations are the same from each actor’s perspective.

**12.7.3 – Measures of Ego Algebras**
Breiger and Pattison (1986) compare ego algebras by the joint right homomorphism comparison of two ego algebras.

**12.7.4 – Example**
The authors used the routine JNTHOM in the program ROLE, and create a complete link hierarchical clustering using the program SYSTAT to calculate ego algebras for advice and friendship relations for Krackhardt’s high-tech managers.

One of the strengths of this approach is that ego algebras can be compared across networks, if the same relations are measured on both groups.

**12.8 – Discussion**
All of the methods that we described in this chapter propose definitions under which actors in a network are to be considered equivalent.

The methods differ in terms of which specific properties are relevant to the equivalence.

If one equivalence definition is more restrictive than another, then any actors who are equivalent by the first definition are also equivalent by the second definition.

The five definitions can be ordered from most restrictive to least restrictive as follows:

- Structural equivalence
- Automorphic or isomorphic equivalence
- Regular equivalence
- Local role equivalence
- Ego algebra equivalence
Protests challenge the polity and their motivation can target policies ranging from tangible social causes (redress discrimination) to broader frames (social justice) to the outrageous (mandatory happy hours).

The “structure” of protests: Named groups protest on named issues. This alignment induces Group-Group relations. Duality implies that relations among groups implies relations among individuals. Basic idea: exploit the inherent duality

What determines success of protest: While protests serve as a resource to gain greater relative power i.e. means to a tangible end, the outcome of their movement is usually determined by the manner of protest, repertoire used, their prevailing image and their adaption to opposition.

This has led to the Resource Mobilization Model that states that success is independent of the number of active groups. In fact, success is dependent on their internal constitution and external support. While the authors are generally sympathetic to this view, they believe that Structural Position and Compositional effects (structural shifts over time) determine success.

The goals of their paper are -
2. Test aspects of the “new social movement theory”
   a. Identity has largely replaced interest
   b. Role of labor has markedly declined over time
3. Dominant repertoires associated with the most central groups

Data & Methods: They included 397 national protests during the key periods executed by named groups from May to September that were reported in The Washington Post (characteristics, target, issue, repertoire used, size & arrests) from 1960 to 1983. Issues were aggregated into domains and GI (Group-issue) matrices were created. Using ordinary (inner product) matrix multiplication of a GI matrix and its transpose GI' (Breiger’s 1974), a GG (Group-Group) matrix was created. The counter position yields an II matrix (Issue to Issue II Matrix).

While it is possible that a newspaper can indulge in selective reporting based on their editorial policies & politics, broader cycles of attention (greater coverage at the start vs end), other sources such as park permits were even less useful. Not all groups were names during all protests, so some were identified by ties. As with all aggregation, data is lost along the way, but blockmodels are more robust

They trace groups through the five key periods.
Peripheral players become central especially if they align with central players. The first period was dominated by the Civil Rights movement, the second by its expansion led by the Southern Christian Leadership Conference, the third by coupling & decoupling, fourth by breakdown of consensus, and the last by return of Labor. The only consistent isolate was the Neo-Nazi group while some groups
isolated themselves based on events (eg. NAACP isolated after assassination of MLK Jr).

The new social movements, which had been previously marginalized are indistinguishable from traditional protest groups. i.e. Identity has replaced interest. But, the role of labor has not declined. Central groups define Repertoire (P1: pickets (Quakers), P2: sit-ins x 3, marches x 2 (SCLC, Quakers), P3: rallies or sit-ins (Radical Left), and P4/P5: rallies & marches). The only interesting exception being the Fundamentalist who shifted from rallies & marches to sit-ins at peak. Later, from the periphery, they experimented with picketing.


Structural data are inherently complex. By implication, our network tools should capture this complexity, but even our state-of-the-art tools are in their infancy. More importantly, our notions of structure are often hazy & conflicting. Borrowing humor from the Nobel Prize winner, Peter Medawar, this prospect makes us solemn, since we ought to be able to define it and shifty-eyed since we often cannot, & don’t want the world to know it. If you give a small boy a hammer, he will find that everything he encounters needs pounding.

Data: 14 political actors from a mid-west county. The substantive issue at hand: Construction of a new jail. Since the county executive wanted to consolidate his power, he proposed eliminating several positions including the auditor’s and was in favor of constructing the new jail. The county auditor became the key player in the oppositional alliance and was against construction of the new jail. To make matters interesting, unlike the current consensus president, the previous county president, a powerful political player played a crucial role due to his connections. The only other non-elected politician, a former council member does not play any role due to his isolate status.

Doreian proposed two Hypotheses - 
H1:Partition. One alliance favors the County Executive while the other alliance favors the County Auditor
H2:Voting will be conditioned by ties among actors

That the majority were White (12/14), male (13/14) & democratic (12/14), further bolstered his claim.

He used several network tools - Multi-Dimensional Scaling (MDS), MDS with adjacency matrix, Centrality Scores, First-order Star (actors directly linked to ego), Robinson’s-a-Measure (vector), MDS: (Euclidean) Kruskal’s Method, MDS: (Euclidean) Guttman-Lingoes Method, MDS: Product Moment Correlations (Faust & Romney), CONCOR (Breiger), Cluster, Cliques, 2-Clique Structure, and Q-analysis (chains of q-connectivity)

The Coarse structure (Alliance A vs B) can be detected by most of the methods, probably in part since it is easier to detect. There was a considerable variation in the Finer structure, which leaves us with blunt tools. Different methods lead to different details which would be acceptable if different structural properties are sought. So what about broader conceptualization? Operationally, we should spell out range of structural & regular equivalence. Pragmatically, we should establish structural properties of interest and use tools based on our analysis of the extent to which they are successful. So, we should select our hammers based on our ability to find appropriate structure!


Definitions
Keiretsu: highly visible clique like patterns based on inter-corporate alliances
Zaibatsu: family-centered holdings from which diversified grouping of firms have descended in the post-war period
Research Question: So financial institutions play a central role in networks of directorships, stock ownership, and corporate control as they do in the U.S.?

Classifying Japanese Firms

Nominal: formal executive participation in monthly meetings for purposes of social & strategic interaction

Structural: measure precisely the characteristics of the relationships among firms in the same keiretsu & compare different groupings

Econometrically: patterns of affiliation to trace performance differences to infer role of groups for member firms

Gerlach considers the entire corporate network as a whole for his starting point of analysis.

Corporate Network Patterns: in the pre-war era, powerful families held a diversified portfolio of corporations that were linked to each other. They also grew their own financial arms/banks as funding sources. In the post-war era, the control became less hierarchical, but every core company retained a similar structure and was surrounded by a major commercial company, a general trading company and a manufacturing firm.

Inter-corporate Alliances: keiretsu groupings based on relatively dense networks of historically determined relationships

Financial Centrality: stratification of network by economic sector & role of financial institutions in capital allocation process

Industrial Inter-dependence: based on business corporations need to manage uncertainty by coordination with competitors and partners

Unlike American laws, banks in Japan are allowed to invest in companies and thus have a vested interest in their success. To spread their risk, a bank will fund companies with interests as diverse as soup-noodles to missiles.

Method: 40 of the largest industrial companies and 20 of the largest financial institutions by assets (10 commercial banks, 5 trust banks & 5 non-life insurance companies) were chosen. Ties along three measures – bank borrowing, equity ownership & director inter-locks were used to create three 60x60 matrices.

Why Blockmodeling?

Capable of addressing each of these varied objectives

Does not impose on the structure a priori categories or attributes

Partitions a network into structurally related groups & derives empirically meaningful patterns of relationships within and between groups

MDS Euclidean Distances & CONCOR were used

Densities among blocks in the directorship matrix are lower than for equity. Equity relationships showed the most complete interlocking. In the debt matrix, 2 bank blocks send nearly all of the ties to all 5 industrial blocks while the third block (non-life-insurance) was a major shareholder but less active in lending. Debt and equity density is associated positively in the two bank blocks.

Majority of ties existed in the three quadrants involving financial institutions while direct ties among the industrial blocks weak or nonexistent. Block assignments do not appear to be based on industrial overlap or on interdependent firms (oil products for chemical production). Block assignments appear to be distributed based on membership in the same keiretsu & geographic location (eg. Osaka)

In summary, Gerlach demonstrates the value of Blockmodel Analysis for corporate networks. American and Japanese corporate structures differ from their inception in history and evolve based on extant legal framework. Keiretsu is seen as an attempt to spread industrial risk, reduce performance variability and share upside gains.