

## PAD 504 - Midterm Test

Please answer each of the following. Put *everything* you want me to see in the bluebooks provided. Show all significant work and any partial answers for part credit. Please do NOT tear out any bluebook pages (other pages tend to fall out); just draw a line through material you want me to ignore.

- (1) [12 points] A student of government finance appropriately objected to the PAD 504 model of the U.S. Federal debt and deficit, on the grounds that the model does not explicitly show debt repayments. Since Treasury bills have maturities of about twenty five years, she argues that about 1/25 of the debt is repaid each year. She also felt the model would be improved if revenues and spending were represented separately. She thinks it would be reasonable to project that revenues grow about 2.5 percent each year.

You decide to satisfy these concerns by creating a revised Federal debt and deficit model that reflects the following assumptions:

$$\begin{aligned} \text{Debt}_t &= \text{Debt}_{t-1} + (\text{Deficit}_{t-1} - \text{Debt repayments}_{t-1}) \\ \text{Deficit}_t &= \text{Spending}_t - \text{Revenue}_t \\ \text{Revenue}_t &= (\text{given in words above}) \\ \text{Spending}_t &= \text{Program spending}_t + \text{Interest payments}_t + \text{Debt repayments}_t \\ \text{Program spending}_t &= \$150 \text{ billion/year (assumed constant here for simplicity)} \\ \text{Interest payments}_t &= \text{Interest rate} * \text{Debt}_t \\ \text{Debt repayments}_t &= (\text{given in words above}). \end{aligned}$$

Create a spreadsheet representation of this Federal debt model, beginning with a layout like the following in your bluebook. Assume a constant interest rate of 7 percent. Also assume the model begins in 1984 with a Federal debt of \$1600 billion and revenues during that year of \$100 billion. Show cell formulas for the next two years.

- (2) [12 points] Translate the following assumptions into a set of five difference equations describing the costs, revenues, and budget surplus of a public sector agency:

Personnel costs in each succeeding year are expected to rise 8 percent per year. Costs of supplies are expected to rise \$120,000 each year. Total costs are the sum of personnel costs and the cost of supplies. Tax revenue is expected to decline 2 per cent each year. The net budget surplus (or deficit) is the difference between revenues and costs.

(Be sure you have an equation for each of the five quantities described.)

- (3) [9 points]

(a) Compute the following matrix product:

$$\begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

(b) The following is a linear difference equation model for transfer patterns among jobs in an agency, along with and new hires, expressed in matrix form:

$$\begin{bmatrix} S_t \\ A_t \\ M_t \end{bmatrix} = \begin{bmatrix} .75 & .08 & 0 \\ .12 & .70 & .05 \\ 0 & .10 & .80 \end{bmatrix} \begin{bmatrix} S_{t-1} \\ A_{t-1} \\ M_{t-1} \end{bmatrix} + \begin{bmatrix} 120 \\ 0 \\ 0 \end{bmatrix}$$

where

$S_t$  = number of staff at time  $t$

$A_t$  = number of middle-level administrators at time  $t$

$M_t$  = number of top-level managers at time  $t$ ,

and the column at the right represents new hires in each of these categories.

Compute  $S_1$ ,  $A_1$ , and  $M_1$ , given that  $S_0 = 700$ ,  $A_0 = 100$ , and  $M_0 = 20$ .

(c) Disregarding new hires, what fractions of Staff (S), Administrators (A), and Managers (M) does the agency *lose* each period? [This does not require extensive computation!]

(4) [12 points] Burtonville's three trash incinerators have the following characteristics:

		Emissions per ton burned :	
Incinerator	Capacity in tons/day	Units of SO2	Units of Particulate

a t o r			
A	1200	250	20
B	800	150	30
C	1000	220	24

Burtonville generates about 4500 tons of trash per day. Working the incinerators at full capacity, Burtonville currently burns about 3000 tons of trash per day and sends the remainder, 1500 tons/day, into a landfill.

New pollution control standards require that sulfur dioxide (SO<sub>2</sub>) emissions and particulate emissions be kept below 400,000 units per day (SO<sub>2</sub>) and 50,000 units per day (particulate). Burtonville wants to decide the amounts of trash to incinerate at each of its incinerators so as to minimize the remainder that has to go to the landfill. Compounding the problem is the fact that the amount of trash that can be sent to each incinerator is limited, as shown in the table above under "Capacity."

Brenda began as follows:

	<u>I</u> <u>n</u> <u>c</u> <u>i</u> <u>n</u> <u>A</u>	<u>I</u> <u>n</u> <u>c</u> <u>i</u> <u>n</u> <u>B</u>	<u>I</u> <u>n</u> <u>c</u> <u>i</u> <u>n</u> <u>C</u>	<u>T</u> <u>o</u> <u>t</u> <u>a</u> <u>l</u> <u>s</u>	<u>C</u> <u>o</u> <u>n</u> <u>s</u> <u>t</u> <u>r</u> <u>a</u> <u>i</u> <u>n</u> <u>t</u>	<u>S</u> <u>i</u> <u>a</u> <u>k</u>
Decision variables:	10 0 0	90 0	80 0			
Landfill = 4500 - total incinerated =				?		
Minimize landfill subject to:						
SO <sub>2</sub>	25 0	15 0	22 0	?	40 0 0 0	?
Particulate	20	30	24	?	?	?
Capacity of A	1	0	0	?	12 0 0	?
Capacity of	0	1	0	?	?	?

B						
Capacity of C	0	0	1	?	10	?
					0	
					0	

Unfortunately, Works unexpectedly quit as Brenda was part way through, so some of the cells (indicated by question marks) are not filled in. Please help Brenda by answering the following:

(a) Write *cell formulas* (without any \$'s please) for all the computations in column E. [E4 is the total sent to the landfill; E6 and E7 are the total amounts of SO2 and particulates generated by the three incinerators; E8, E9, and E10 are the total amounts of material burned at each of the three incinerators.]

(b) Provide (compute if necessary) the *quantities* that should appear in all the cells with question marks. [Please *label* each quantity by its cell, e.g., E4 = \_\_\_\_\_ .]

(c) Assuming the spreadsheet has been set up and filled in properly, explain *how you would use it* to solve Burtonville's optimization problem. [What, precisely, would you do? What would you be concerned about? How would you know when you were done?]

Year			
Debt			
Interest rate			
Interest payment			
Debt repayments			
Pgm spending			
Total spending			
Revenues			
Deficit			