I. Review of Spells1
   A. What observed?
   B. Answers to assignment questions in (1) and (2)
   C. Conditions that establish equilibrium
      1. Inflow = outflow
      2. so Inflow = Stock/Time constant
      3. so equilibrium Stock = Inflow * Time constant
   D. So equilibrium in Spells1 and Spells2:
      1. Slow movers in poverty = New SMs in poverty * SM time in poverty
      2. Fast movers in poverty = New FMs in poverty * FM time in poverty
      3. Slow movers within one year of poverty = SMs moving out of poverty * One year
      4. Fast movers within one year of poverty = FMs moving out of poverty * One year

II. Review of Spells2
   A. What observed?
   B. Answers to assignment questions in (3) (other values of the New fraction.
   C. Discussion of (4)
      1. Design of experiments
      2. Observations
      3. Conclusions

III. Analytic equilibrium: Spells3
   A. Write initial values as in I, D.
   B. Simulations showing initial equilibrium no matter what the values of the constants are.
   C. Bane and Ellwood (1985) conclusions:
      “...We found that most of those who every become poor will have only a short stay in poverty.
At the same time, the majority of people who are poor at a given time will have very long
spells of poverty before they escape. These findings suggest ... that most of the people helped
by programs to aid the economically disadvantaged use them only briefly. But the bulk of
resources almost certainly go to a much small group of people who have very long stays in
poverty. The policy dilemmas that this finding poses are serious indeed.”

II. Introduction to Modeling
   A. The dynamics of spreading a rumor - a sequence of models
   B. Model 1 - Wise, Ignorant, and a constant Rumor Spreading Rate
      1. Stock-and-flow diagram
      2. Parameters
         a) initial values: Wise = 2, Ignorant = 1000 (or 998)
         b) value of the Rumor Spreading Rate = 20 people/day
      3. Computing with algebra: Wise(t) = Wise(0) + time*RSR
      4. Discuss flaw in the Rumor Spreading Rate
a) Constant would make Wise negative after a while
b) So RSR must change, but that would mean we could not compute with algebra
5. So plan on slicing time up into intervals of length DT and computing Wise(t) = Wise(t-DT) + DT*RSR
   a) Same form as algebraic computation
   b) But lets RSR vary between successive values of DT

C. Model 2 - a varying Rumor Spreading Rate
   1. Get suggestions for desirable characteristics of the rate
   2. Diagram what is desirable (positive loop and negative loop)
   3. Formulate RSR = constant*Wise*Ignorant
   4. Hand simulate with DT = .5
   5. Spreadsheet simulation (Rumor spreadsheet saved on disk)

III. Introduction to Vensim
   A. Model 2 in Vensim
      1. Build the model
      2. Simulate with various DT's
      3. Show all of Vensim’s tricks
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IV. Population models, introducing the behavior of first-order systems, both linear and nonlinear

A. Births per year constant: linear growth

![Diagram of population growth with births per year and population inputs]

1. Constant births per year

B. Births per year as a fraction of population: a feedback loop

1. Set \( P(0) = 1000 \), constant = 0.05

![Diagram of population growth with births per person per year and population inputs]

2. Note exponential growth (if the constant is positive)

3. Vary the constant: 0.03, 0.05, 0.07, 0.10. Determine doubling times by inspection.

4. State the "rule of 70" for doubling times:
   a) (Percent growth factor) \( \times \) (Doubling time) \( \approx 70 \)
   b) or Doubling time \( \approx 70 \) / (percent growth factor)
   c) or (Fractional growth factor) \( \times \) (Doubling time) \( \approx 0.7 \)

C. The mathematics of exponential growth

1. Derive \( P(t) = P(0) * b^{kt} \)
   a) \( P(dt) = P(0) + dt *[k*P(0)] = P(0) (1 + dt*k) \)
   b) \( P(2dt) = P(dt) + dt *[k*P(dt)] = P(dt) (1 + dt*k) = P(0) (1 + dt*k)^2 \)
   c) \( P(n*dt) = P(0) (1 + dt*k)^n \)
   d) Let \( t = n*dt \), so \( P(t) = P(0) (1 + dt*k)^{t/dt} \)
   e) \( P(0) [(1 + dt*k)^{1/dt}]^{kt} = P(0) b^{kt} \)

2. Note that \( b = (1 + dt*k)^{1/dt} \) approaches \( e \approx 2.71828 \) as \( DT \) gets smaller.

3. Use \( P(t) = P(0) \exp(kt) \) to get rule for doubling times
   a) Doubling time is \( T \) where \( P(T) = 2 P(0) \)
   b) Solve \( P(0) \exp(kT) = 2 P(0) \) for \( T \)
   c) \( P(0) \) cancels, so \( \exp(kT) = 2 \), which by definition of \( \exp \) and \( \log \) means \( kT = \log(2) \), or \( T = \log(2)/k \).
   d) Since \( \log(2) \approx 0.693 \approx 0.7 \), we have the rule of 0.7 or 70%

D. Population and deaths

1. Constant average lifespan: a negative feedback loop
2. Set \( P(0) = 1000 \), average lifespan = 50

3. Note this loop as a causal loop

4. Note half-life satisfies \((1/\text{lifespan}) \times \text{half-life} \approx 0.7\) — same rule as doubling times. [The reason is that in the derivation one gets two negatives (one for outflow and one for \( \logn(1/\text{lifespan}) \)) that cancel out so the algebra is almost exactly the same.]

E. Births and deaths together

1. Behavior depends on which loop is stronger
   a) Compare base run (constants = 0.05 and 50) with birth factor = 0.01.
   b) Loop dominance

F. Population growth in a limited environment
   1. Add carrying capacity and a nonlinear effect of population density on average lifespan
   2. Parameters: 0.05, \( CC = 5000 \), lifespan table = 70, 70, 68, 50, 30, 18, 15.
3. Average lifespan

4. Use strip graphs to track down the sources of the behavior

5. Population
   - 6,000
   - 4,500
   - 3,000
   - 1,500
   - 0

6. Deaths per year
   - 400
   - 300
   - 200
   - 100
   - 0

7. Note as density increases, average lifespan decreases, which increases deaths per year. When the negative loop becomes dominant, the behavior shifts from steepening growth to flattening growth — S-shaped growth.

V. Homework preview

A. Simple urban models
   1. A first order model for Business Structures in a city
   2. A two-level model involving both Business Structures and Population and their interactions: jobs, employment and unemployment (in a ratio of the Laborforce to Jobs), and migration.

B. Purposes:
   1. Working with Vensim
   2. Gaining insights into the behavior of simple nonlinear systems.
3. Thinking about real cities as you work through the various simulation runs — try to think of parameter changes as representing policy or scenario changes in a city.

4. Leading to a three-level model of urban dynamics that exhibits the overshoot and decline characteristic of maturing cities in the U.S. (Assignment 4).

C. Vensim details

1. Custom graphs showing selected variables can be created by clicking on the Control Panel button and selecting Graphs in the control panel:

2. Click in New in the Graph control panel opens the following sort of graph definition window. One selects the variables to appear using the Sel buttons. An X in the scale box on the left between two (or more) variables means they will be plotted on the same scale. Vensim will chose its own scales unless you enter values for the scales on the right.